Tempearature and Heat

Temperature and state coordinate:
In the study of mechanical systems, only three fundamental quantities have been needed- length, mass, and time. All other mechanical quantities such as force, energy, and momentum can be expressed in terms of these three quantities. We now consider a class of phenomena called thermal or heat phenomenon that requires a fourth fundamental quantity, called temperature.

Temperature can be defined as the quantity which determines whether a body will felt hot or cold, worm or cool to touch.

In thermal phenomena, in any system the quantities such as length of an object, the volume, pressure of liquid or gas or resistance of a body changes with the change of temperature and is known as the **state coordinate**. Therefore L, V, P, R, etc are state coordinates of the systems.

Thermal equilibrium:
If a body A which is felt hot and another body B is felt cold are placed in contact with each other, after a sufficient length of time A and B give rise to the same temperature sensation; then A and B are said to be in thermal equilibrium.

Zeroth law of thermodynamics:
According to zeroth law of thermodynamics, *two systems in thermal equilibrium with a third are in thermal equilibrium*. If A and B are separately in thermal equilibrium with a third body C then A and B are in thermal equilibrium with each other.

Another statement of the zeroth law is, *there exists a scalar quantity called temperature which determines whether two bodies are in thermal equilibrium or not*. The essence of the zeroth law is the existence of a scalar quantity called temperature.

Phase equilibrium:
A material can exist in three phases namely solid, liquid, and gas. Only at a certain definite temperature it is possible to exist two of the phases (solid & liquid, or liquid & gas, or solid gas) together without changing their own phase. This condition is known as phase equilibrium and is possible only if pressure is constant.

Melting point:
The temperature at which a solid and liquid of the same material coexist in phase equilibrium at atmospheric pressure is called melting point. For water the melting point is $0^{\circ}C$.

Boiling point:
The temperature at which a liquid and vapour of the same material coexist in phase equilibrium at atmospheric pressure is called boiling point. For water it is $100^{\circ}C$.

Sublimation point:
If the solid and vapour phase are coexist at a definite temperature, it is called sublimation point.

Triple point of water:
It is possible for all three phases of water to coexist in equilibrium only at a definite pressure and temperature, known as triple point. The pressure of triple point of water is $610Pa$ or $4.58mm\ Hg$ and the temperature is $0.01^{o}C$ or $273.16K$.

Different temperature scales:
Thermometer is used to measure the temperature. There are three well known scales for measuring temperature, namely Celcius, Rankine and Farenhite scale. They are related as

$$\frac{C}{5}=\frac{F-32}{9}=\frac{R}{4}$$

There is another scale of temperature which is adopted as SI unit of temperature. This is Kelvin scale and is defined as

$$0^{o}K = -273.15^{o}C \text{ or } 0^{o}C = 273.15^{o}K$$

Definition of heat:
Heat is a form of energy which is transferred between a system and its surroundings because of the temperature difference only.

Units of quantity of heat:
In the eighteenth century, a unit of quantity of heat, the **calorie** was defined as the amount of heat required to raise the temperature of 1gm of water $1K$ or $1^{o}C$. It was found later that more heat was required to raise the temperature of $1gm$ of water from $90^{o}C$ to $91^{o}C$ than from $30^{o}C$ to $31^{o}C$.
The definition was then refined, and the chose calorie became "$15^{\circ}C$ *calorie*", that is "the quantity of heat required to change the temperature of $1gm$ of water from $14.5^{o}C$ to $15.5^{o}C$".
A corresponding unit defined in terms of Fahrenheit degrees and British units is the **British thermal units**, or BTU.
By definition, $1BTU$ is the quantity of heat required to raise the temperature of 1lb of water from $63^{o}F$ to $64^{o}F$.
A third unit in common use, especially in measuring food energy, is $kcal$. The relations among these three units are

$$1BTU = 252cal = 0.252kcal$$

Heat capacity:
The heat required to change the temperature of a body a unit amount is defined as the heat capacity of that body. It is denoted by C.

$$\therefore C=\frac{\Delta Q}{\Delta T} \qquad \text{............ (1)}$$

unit: The SI unit of heat capacity is JK^{-1}.

Specific heat capacity:
The heat required per unit temperature rise per unit mass of the body is known as the specific heat capacity. It is denoted by c.

$$\therefore c=\frac{dQ}{mdT} \qquad \text{............ (2)}$$

where m is the mass of the body.
unit: The SI unit of specific heat capacity is $Jkg^{-1}K^{-1}$.
To calculate the quantity of heat, we have from (2)

$$dQ = mcdT$$

Integrating, we get

$$Q = mc\int_{T_1}^{T_2} dT$$

$$\therefore Q = mc(T_2 - T_1)$$

This is the quantity of heat to change the temperature from T_1 to T_2 of a body whose mass is m and specific heat capacity is c.

Change of phase:

We know any matter can exist in three phases‒ solid, liquid and gas; and at a particular temperature two of their phase can exist without changing their phase. Above this temperature the phase is changed.

For water at 0^oC the solid and liquid phase can coexist. As soon as this temperature is reached, the ice begins to melt and temperature remains 0^oC until all the ice has been melted. The heat required to melt all the ice at a fixed temperature is known as the **heat of fusion of ice** and is measured as the unit of Jkg^{-1}. For ice, the heat of fusion is $3.34\times10^5\,Jkg^{-1}$.

At 100^oC the liquid water and water vapor can coexist and if the heat is being supplied continuously after reaching 100^oC the liquid water starts to convert into vapor until all the water has been converted.

The heat required to convert the liquid into its vapor at a constant temperature (boiling point) is known as **heat of vaporization**. For water the heat of vaporization is 2.26×10^6 Jkg^{-1}.

The temperature remains constant during each change of phase, providing the pressure remain constant.

Heat transfer:

There are three processes of transferring heat energy form one place to another namely

i) Conduction
ii) Convection
iii) Radiation

The processes are discussed below briefly.

Conduction:

If heat is transferred through a medium (or a body) only due to the energy transfer from one particle to other particle of the medium; that is, if the particles of the medium do not take part in transferring process then the process is known as conduction. In solid bodies the heat is transferred by conduction process.

We can calculate the amount of heat which is transferred from one end to other of a body of length L and are A. Experiment shows that for a body with uniform cross section the heat transfer per unit time is directly proportional to the area of the body, temperature difference between two ends of the body and inversely proportional to the length of the body. i.e.

$$H \infty A$$

$$H \infty (T_2 - T_1)$$

$$H \infty \frac{1}{L}$$

$$\therefore H = \frac{KA(T_2 - T_1)}{L}$$

where K is a proportionality constant known as **thermal conductivity of the body**. The quantity H is the heat transferred per unit time is known as the **heat current**.
If the cross section is not uniform then this expression can be written as

$$H = -KA\frac{dT}{dx}$$

The quantity dT/dx is known as **temperature gradient**. The *(-ve)* sign indicates that if T increases with x then the direction of heat flow is the direction of decreasing x.

Convection:
Convection is process for transfer of heat by actual motion of material; that is, if the particles of the medium take part in the transferring process the process is then called convection. In liquids the heat is transferred by convection process.

Radiation:
If heat is transferred without any medium the process will be called radiation. We get heat from the sun by radiation process.

Stefan-Boltzmann law:
The rate of radiation of energy from a surface is proportional to the surface area and to the fourth power of the absolute temperature T. It also depends on the nature of the surface, described by a dimensionless number e, which in between 0 and 1.

$$H \infty AeT^4$$

$$\therefore H = Ae\sigma T^4$$

where $\sigma = 5.67\times10^{-8} Wm^{-2}K^{-4}$ is universal physical constant called the Stefan-Boltzmann constant. e characterizing the emitting properties of a particular surface, is called the **emissivity**.

Exercise 15-8: A calorimeter contains $100gm$ of water at $0°C$. A $1000gm$ copper cylinder and a $1000gm$ lead cylinder, both at $100°C$, are placed in the calorimeter. Find the final temperature if there is no loss of heat to the surroundings. The specific heat capacity of water, copper, and lead are $4.19Jgm^{-1}°C^{-1}$, $0.47Jgm^{-1}\ °C^{-1}$ and, $0.13Jgm^{-1}\ °C^{-1}$, respectively.

Solution:
Given that

$$m_{H_2O} = 100gm, m_{Cu} = 1000gm, m_{Pb} = 1000gm, \theta_1 = 0°C, \theta_2 = \theta_3 = 100°C$$

$$c_{H_2O} = 4.19Jgm^{-1}°C^{-1}, c_{Cu} = 0.47Jgm^{-1}°C^{-1}, c_{Pb} = 0.13Jgm^{-1}°C^{-1}$$

Let us suppose that the final temperature is θ_f.
Heat absorbed by water $Q_1 = m_{H_2O}c_{H_2O}(\theta_f - \theta_1) = 100\times4.19\times(\theta_f - 0)$
Heat released by copper $Q_2 = m_{Cu}c_{Cu}(\theta_2 - \theta_f) = 1000\times0.47\times(100 - \theta_f)$
Heat released by lead $Q_3 = m_{Pb}c_{Pb}(\theta_3 - \theta_f) = 1000\times0.13\times(100 - \theta_f)$
Since,

Heat released = Heat absorbed

$$Q_2 + Q_3 = Q_1$$

$$\Rightarrow 1000\times0.47\times(100 - \theta_f) + 1000\times0.13\times(100 - \theta_f) = 100\times4.19\times(\theta_f - 0)$$

$$\therefore \theta_f = 55.86°C$$

Exercise 15-10: An aluminium can of mass $500gm$ contains $117.5gm$ of water at a temperature of $20°C$. A $200gm$ block of iron at a temperature of $75°C$ is dropped into the can. Find the final temperature assuming no heat loss to the surroundings. The specific heat capacity of water, aluminium, and iron are $4.19 Jgm^{-1}°C^{-1}$, $0.91 Jgm^{-1}°C^{-1}$ and, $0.47 Jgm^{-1}°C^{-1}$, respectively.

Solution:

Given that

$$m_{H_2O} = 117.5gm, m_{Al} = 500gm, m_I = 200gm, \theta_1 = \theta_2 = 0°C, \theta_3 = 75°C$$

$$c_{H_2O} = 4.19Jgm^{-1}°C^{-1}, c_I = 0.47Jgm^{-1}°C^{-1}, c_{Al} = 0.91Jgm^{-1}°C^{-1}$$

Let us suppose that the final temperature is θ_f.

Heat absorbed by water $Q_1 = m_{H_2O}c_{H_2O}(\theta_f - \theta_1) = 117.5 \times 4.19 \times (\theta_f - 20)$

Heat absorbed by aluminium $Q_2 = m_{Al}c_{Al}(\theta_f - \theta_2) = 500 \times 0.91 \times (\theta_f - 20)$

Heat released by iron $Q_3 = m_I c_I(\theta_3 - \theta_f) = 200 \times 0.47 \times (75 - \theta_f)$

Since,

Heat released = Heat absorbed

$$\Rightarrow Q_3 = Q_1 + Q_2$$

$$\Rightarrow 117.5 \times 4.19 \times (\theta_f - 20) + 500 \times 0.91 \times (\theta_f - 20) = 200 \times 0.47 \times (75 - \theta_f)$$

$$\therefore \theta_f = 24.96°C$$

Exercise 15-13: A copper calorimeter of mass $300gm$ contains $500gm$ of water at a temperature of $15°C$. A $560gm$ block of copper at a temperature of $100°C$ is dropped into the calorimeter and the temperature is observed to increase to $22.5°C$. Find the specific heat capacity of copper. The specific heat capacity of water is $4.19Jgm^{-1}°C^{-1}$.

Solution:

Given that

$$m_{Cu} = 300gm, m_{H_2O} = 500gm, m_{Cu-block} = 560gm, \theta_1 = \theta_2 = 15°C, \theta_3 = 100°C$$

$$\theta_f = 22.5°C, c_{H_2O} = 4.19Jgm^{-1}°C^{-1}$$

Heat gained by calorimeter $Q_1 = m_{Cu}c_{Cu}(\theta_f - \theta_1) = 300 \times c_{Cu} \times (22.5 - 15)$

Heat gained by water $Q_2 = m_{H_2O}c_{H_2O}(\theta_f - \theta_2) = 500 \times 4.19 \times (22.5 - 15)$

Heat lost by copper $Q_3 = m_{Cu-block}c_{Cu}(\theta_3 - \theta_f) = 560 \times c_{Cu} \times (100 - 22.5)$

Since,

Heat lost = Heat gained

$$\Rightarrow Q_3 = Q_1 + Q_2$$

$$\Rightarrow 560 \times c_{Cu} \times (100 - 22.5) = 300 \times c_{Cu} \times (22.5 - 15) + 500 \times 4.19 \times (22.5 - 15)$$

$$\therefore c_{Cu} = 0.382Jgm^{-1}°C^{-1}$$

Problems for practice: Exercise 15- 14, 15- 26, 16- 01, 16- 03, 16- 09, 16- 12, 16- 21, 16- 26, 16- 27.