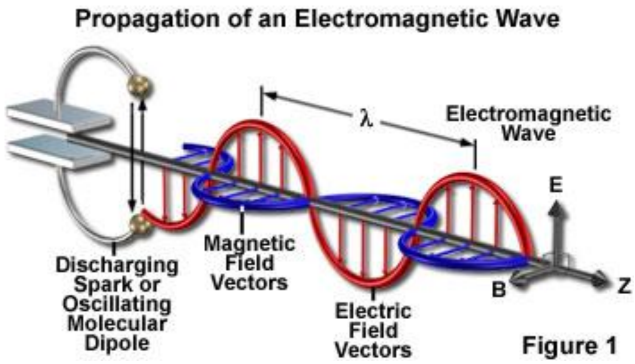
Quantum Theory of Radiation

Introduction: In 1864 British physicist James Clerk Maxwell made suggestion that accelerated electric charges linked electric and magnetic disturbances that can travel indefinitely through space. If, the charges oscillate periodically, the disturbances are waves whose electric and magnetic components are perpendicular to each other and to the direction of motion.

James Clerk Maxwell (1831–1879)



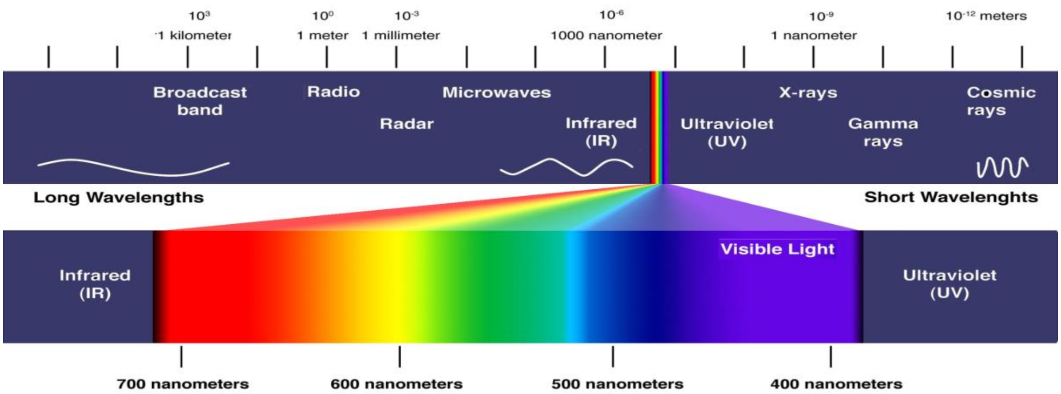
Thus electromagnetic wave nature of light was predicated by Maxwell and demonstrated experimentally by Hertz. This fact was further supported by explanation of the phenomena such as interference, diffraction, polarization on the basis of Huygens wave theory. However, the experimentally observed phenomena such as Compton effect, the spectrum of black body radiation and the characteristic spectra of atoms could not be explained on the basis of and Huygens wave theory. These phenomena, concerned with the emission and absorption of radiation by matter, indicated the existence of energy quanta.

Electromagnetic waves can be regarded as waves because under suitable circumstances they exhibit diffraction, interference, and polarization. Similarly, under other circumstances electromagnetic waves behave as though they consist of streams of particles.

Maxwell was able to show that the speed of electromagnetic wave in free space is given by

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 2.998\times10^{8} \text{ m/s.}$$

where ε_0 is the electric permittivity of free space and μ_0 is the magnetic permeability. This is the same as light waves.

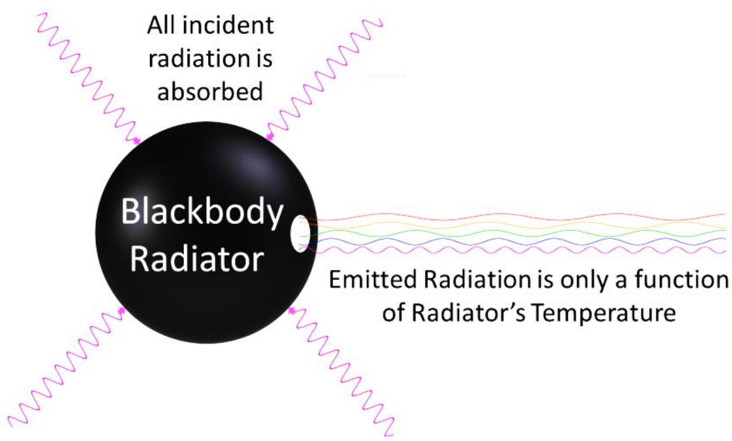


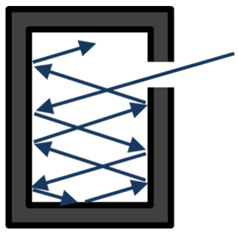
When a piece of iron rod is ignited it shows the following transition

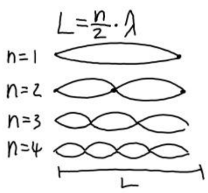
In fact, other frequencies to which our eyes do not respond are present as well. All objects radiate energy continuously whatever their temperature is. At room temperature most of the radiation is in the infrared part of the spectrum and hence invisible.

Black body radiation and Rayleigh-Jeans formula

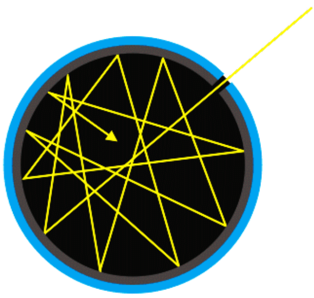
The ability of a body to radiate is closely related to its ability to absorb radiation. A body in thermal equilibrium with its surroundings absorbs energy at the same rate as it emits energy. An ideal body that absorbs all the radiation incident upon it, regardless of frequency is called a blackbody.



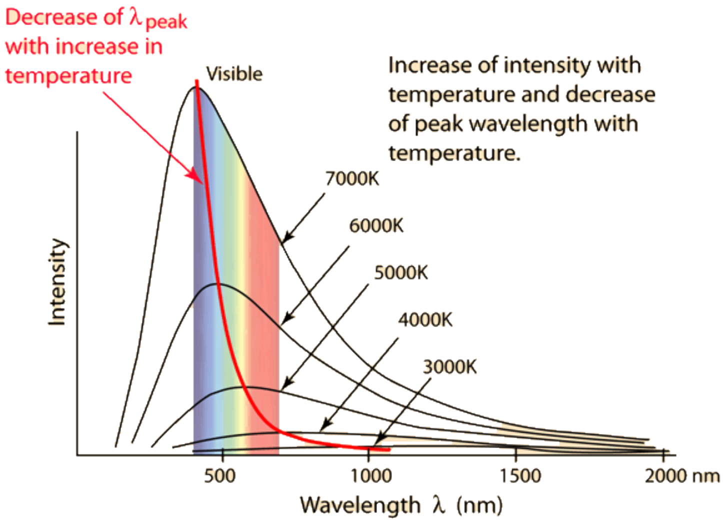




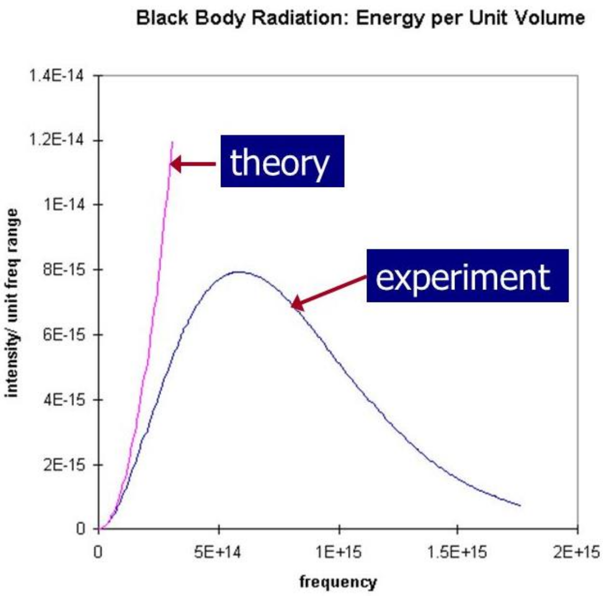
A hole in the wall of a hollow object is an excellent approximation of blackbody.

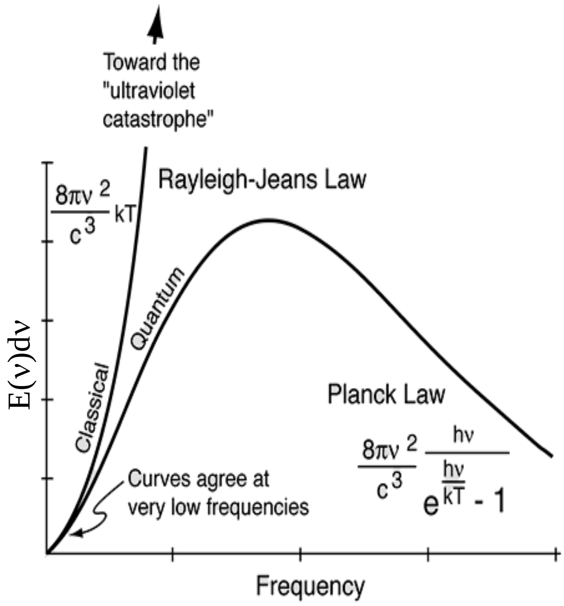


Conceptual Black Body



Electromagnetic radiation in the cavity whose walls are perfectly reflecting consists of standing waves that have nodes at the walls which restricts their possible wavelengths (**Quantization of EM waves**).





The total energy E(ν)dν of per unit volume in the cavity in the frequency interval from ν and (ν+dν) is $E(\nu)d\nu = \frac{8\pi kT}{c^3}\nu^2 d\nu$ Rayleigh-Jeans Formula.

It shows, as ν→∞, E(ν)dν→∞. But, in reality as ν→∞, E(ν)dν→0.

This discrepancy is known as **ultraviolet catastrophe** of classical physics.

Planck's Radiation formula for the interpretation of quantum theory:

Planck consider a formula for the radiation like

$$E(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{\frac{h\nu}{kT}}-1}d\nu$$

Where h is the Planck's constant and $h = 6.626\times10^{-34}$ J.S

At higher frequencies, hν >> kT and therefore, $e^{\frac{h\nu}{kT}} \rightarrow \infty$. That is E(ν)dν → 0.

At lower frequencies, hν << kT and $\frac{h\nu}{kT} << 1$. Therefore, $e^{\frac{h\nu}{kT}} \approx 1 + \frac{h\nu}{kT}$.

Therefore, $E(\nu)d\nu \approx \frac{8\pi h}{c^3}\frac{\nu^3}{1+\frac{h\nu}{kT}-1}d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{\frac{h\nu}{kT}}d\nu = \frac{8\pi kT}{c^3}\nu^2 d\nu$ → Rayleigh-Jeans Formula.

At lower frequencies, Planck's Radiation formula →Rayleigh-Jeans formula and thus valid both for lower and higher frequency region and the ultraviolet catastrophe is removed.

He proposed the oscillator energy will be $E_n = nh\nu$, where n = 0, 1, 2, …. any integer.

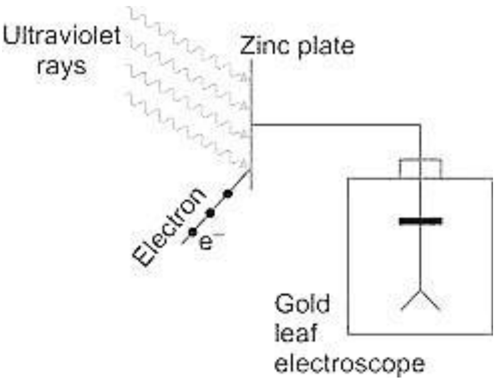
That is, an oscillator emits radiation of frequency, ν, when it drops form one energy state to the next lower one and it jumps to the next higher state when it absorbs radiation of frequency, ν. Each discrete bundle of energy, $h\nu$ is called a **quantum** from a Latin word meaning "how much".

Average oscillator energy per standing wave therefore is not $\bar{\epsilon} = kT$, instead it is $= \frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$, which leads to the Planck's radiation formula.

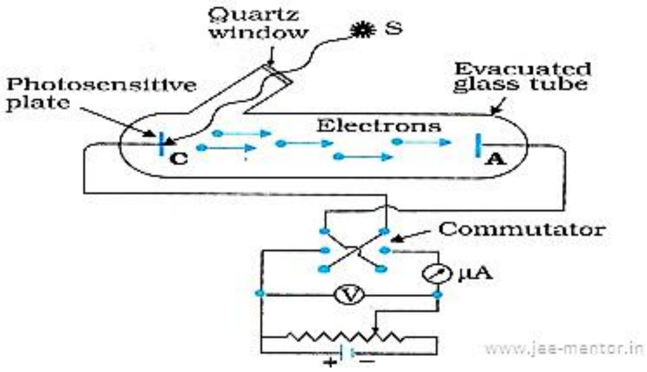
Photoelectric effect: Latest in the laptop

The emission of electron form the surface of a material, when illuminated by electromagnetic radiation of frequency higher than a specific value is known as photoelectric effect, and the emitted electron is termed as a photoelectron.

Hertz first observed this effect in 1887, accidentally, when he noticed that sparks occurred more readily in the air gap of two electrically changed spheres when ultraviolet light was directed, but was thoroughly investigated by W. Hallwachs and Phillip Lenard during the last decade of 19^{th} century.

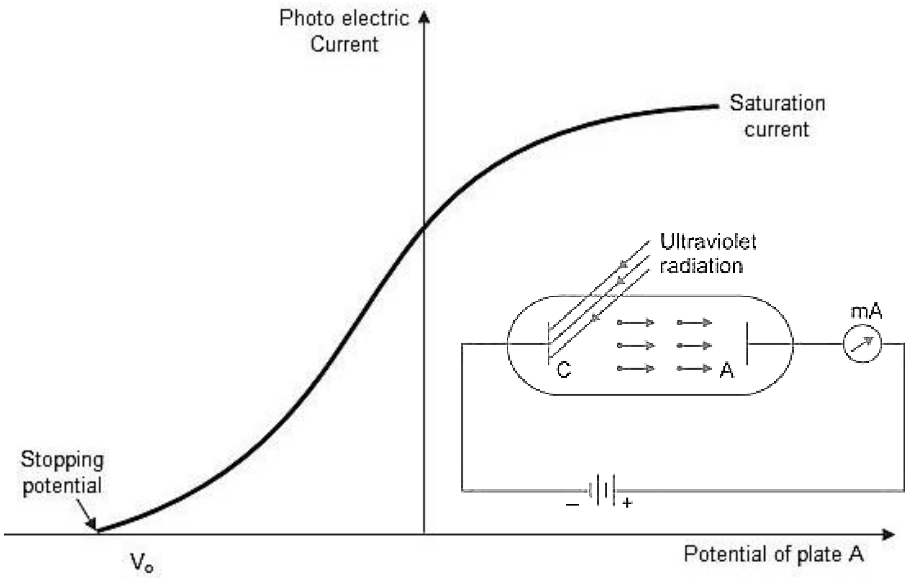


Hertz and Lenard's observations



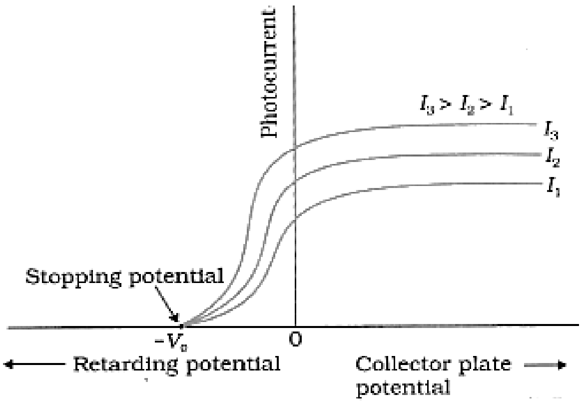
Photoelectric effect

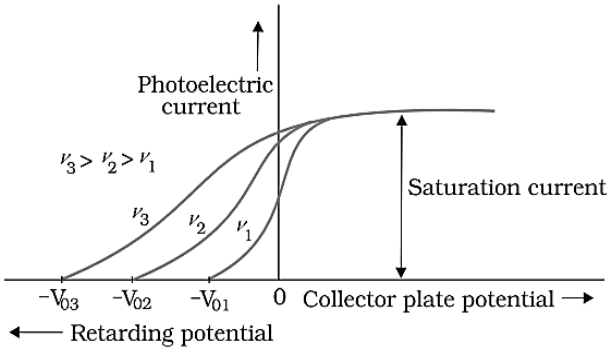
Fig shows how the photoelectric effect was studied. An evacuated tube contains two electrodes connected to a source of variable voltage with the metal plate whose surface is irradiated as anode.

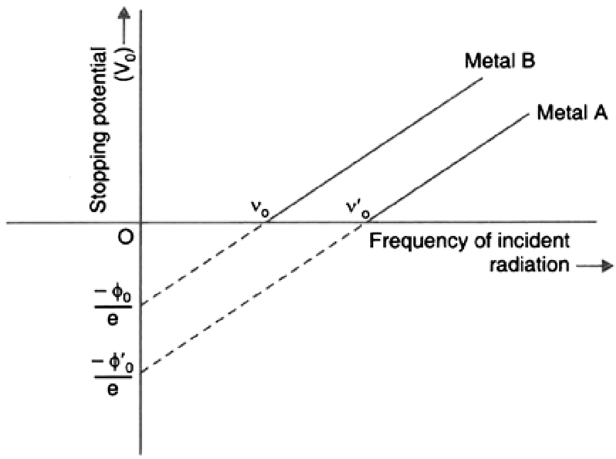


Some of the photoelectrons that emerge from the metal surface have enough energy to reach the cathode despite its negative polarity, and they constitute the measured current. As long as the voltage is increased to a higher negative voltage, V_0, no more photoelectrons achieve, as indicated by current dropping to zero. This potential is known as stopping potential.

Experimental facts and the classical theory:







Since, Light waves carry energy, some of the energy absorbed by the metal may somehow concentrated on individual electrons and reappear as their kinetic energy. But three experimental findings show that no such simple explanation is possible.

1. Within the limit of experimental accuracy there is **no time lag** between the arrival of metal surface and the emission of photo electrons. According to classical theory, a period of time should elapse before an electron accumulates enough energy (several eV) to leave the metal, because the energy in an electro-magnetic wave is supposed to be spread across the wave fronts. Calculation predicts for Na surface of 1 atom thick and area 1 m^2 will need over a month.

2. A bright light yields more photoelectrons than that one of the same frequency, but the electron energies remain the same. The **electro-magnetic theory** of light, on the contrary predicts that the **more intense** the light, the **greater the energies** of the electrons.

3. The higher the frequency of the Light, the more energy the photoelectrons wave.

4. At frequencies below a certain critical frequency ν_0, which is the characteristic of each particular metal, no electrons are emitted. Above ν_0, the photoelectron energy increases linearly with increasing frequency. This cannot be explained by the electro-magnetic theory of light

Quantum view of photoelectric effect:

In 1905 Einstein realized that the photoelectric effect could be understood if the energy in light is not spread out over wave fronts but is concentrated in small packets or photons. Each photon of light of frequency ν has energy $h\nu$, the same as Planck's quantum energy.

Einstein's break with classical physics was more drastic. Energy is only given to electro-magnetic waves in separate quanta. On this basis this hypothesis of Einstein results of the photoelectric effect is explained as follows:

1. Because electro-magnetic wave energy is concentrated in photons and not spread out, there should be no delay in the emission of photoelectrons.

2. All photons of same frequency have the same energy. So changing the intensity of a monochromatic light beam will change the number of photoelectrons but not their energies.

3. The higher the frequency, the greater the photo energy $h\nu$ and so the more energy the photoelectrons have.

4. There must be a minimum energy ϕ for an electron to escape from a particular metal surface or else electrons will pour out all the time. The energy is called the work function of the metal and related to by the formula, $\phi = h\nu_0$
The greater the work function of a metal, the more the energy is needed for an electron to leave from a metal surface.

Einstein's Photoelectric Equation:

According to Einstein, the photoelectric effect in a given metal should obey the equation

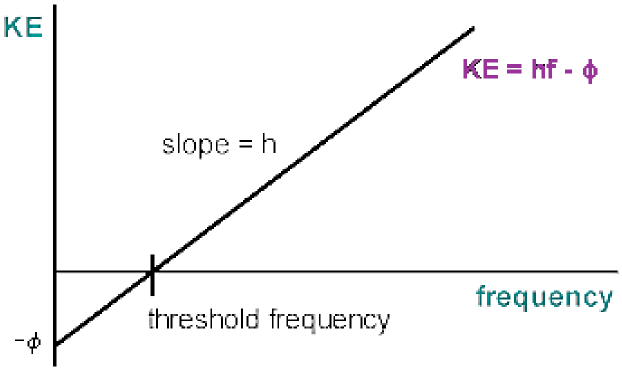
$$h\nu = KE_{max} + \phi$$

Where KE_{max} = maximum photoelectron energy, and $h\nu_0$ = minimum energy needed for an electron to leave the metal. Therefore,

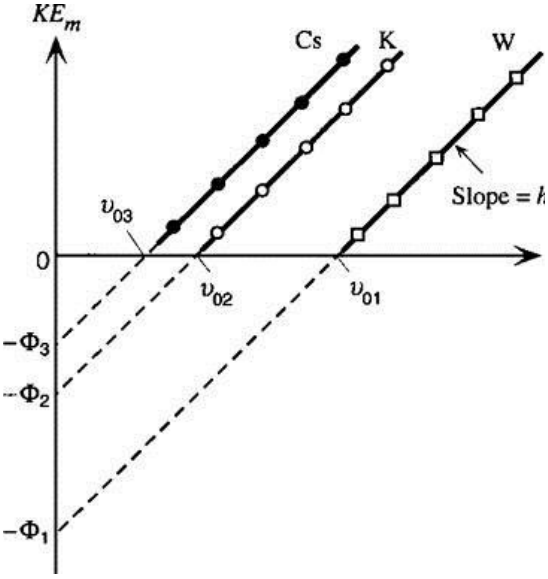
$$KE_{max} = h\nu - h\nu_o = h(\nu - \nu_o)$$

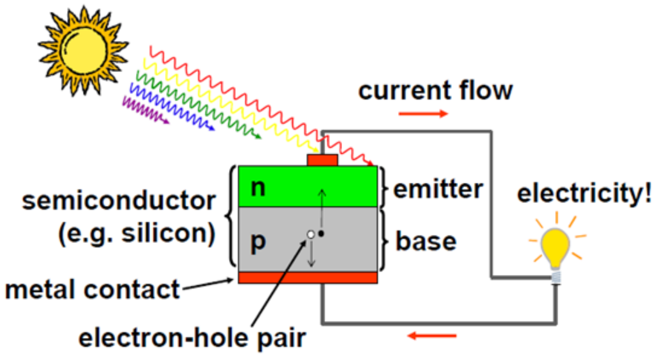
Or, $\frac{1}{2}mv_{max}^2 = h\nu - h\nu_o$

Or, $eVs = h\nu - h\nu_o$



If Einstein was right, the slopes of the lines should be equal to Planck's constant.





Solar (or photovoltaic) **cells** convert the sun's energy into electricity. Whether they're adorning your calculator or orbiting our planet on satellites, they rely on the **photoelectric effect**: the ability of matter to emit electrons when a light is shone on it.

Example:

1. Ultraviolet light of wavelength 350 nm and intensity 1.00 W/m^2 is directed at a potassium surface (work function 2.2 eV), (a) Find the maximum KE of photoelectrons. (b) If 0.50 percent of the incident photons produce photoelectrons how many electrons are emitted per second if the potassium surface has an area of 1 cm^2?
2. The threshold wavelength for photoelectric emission in tungsten is 230 nm. What wavelength of light must be used in order for electrons with a maximum energy of 1.5 eV to be ejected?
3. The threshold frequency for photoelectric effect in copper is 1.1×10^{15} Hz. Find the maximum energy of photoelectrons (in eV) when light of frequency 1.5×10^{15} Hz is directed on a copper surface. Ans: 2.65×10^{-19} J or 1.66eV
4. What is the maximum wavelength of light that will cause photoelectrons to be emitted form Na? (ϕ_{Na}=2.2 eV). What will be maximum KE of photoelectrons be if 200 nm light falls on a sodium surface?
5. A metal surface illuminated by 8.5×10^{14} Hz light emits electrons whose maximum energy is 0.52 eV. The surface illuminated by 12.0×10^{14} Hz light emits electrons whose maximum energy is 1.97 eV. From these data find the value of h and ϕ of the surface.
6. The ϕ of W surface is 5.4 eV. When surface is illuminated by light of λ = 175 nm, the KE_{max} of the photoelectron is 1.7 eV. Find the value of Planck's constant from these data. Find the stopping potential also.

Compton Effect

According to quantum theory– ***photon behaves like particles except for their lack of vest mass***.

When an X-ray photon strikes an electron assumed to be initially at rest in the Laboratory coordinate system and is scattered away from its original direction of motion, while the electron receives an impulse and begins to move. The amount of energy that is lost by the photon in the collision will appear as kinetic energy of the electron, although separate photons are involved. This scattering of a photon by an electron is called Compton Effect.

In such a situation, let initial photon has the frequency ν associated with it, the scattered photon has the lower frequency ν' where

loss in photon energy = gain in electron energy

$$h\nu - h\nu' = KE \qquad \rightarrow (1)$$

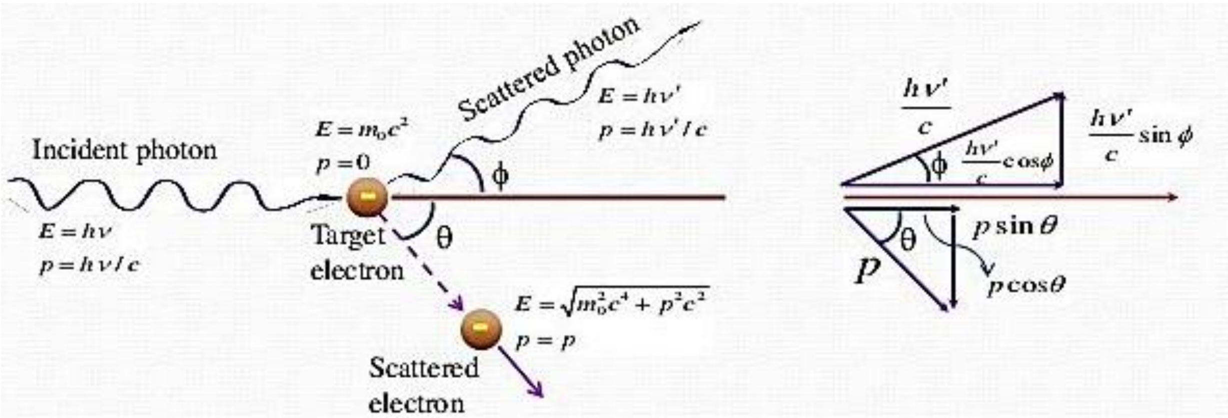
Now, we know momentum of photon (mass less particle) is related to its energy by the formula.

$$E = pc$$

$$p = \frac{E}{c} = \frac{h\nu}{c} \qquad \rightarrow (2)$$

Momentum, is a vector quantity and thus in the collision momentum must be conserved in each of two mutually perpendicular direction. In figure these two directions are– one is the direction of the original photon and other is the perpendicular to it in the plane containing the electron and the scattered photon.

The initial photon momentum is $h\nu/c$, scattered photon momentum is $h\nu'/c$ and the initial and final electron momenta are '0' and p, respectively.



In Compton scattering, energy and momentum are conserved and as a result the scattered photon has less energy than the incident photon. Figure shows the Compton effect and the vector diagram of the momenta and their components of the incident and scattered photons and electron.

In the original photon direction,

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta \qquad \rightarrow (3)$$

And perpendicular to this direction

$$0 + 0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta \qquad \rightarrow (4)$$

Here ϕ is the angle between the directions of the initial and scattered photons and θ is that between the directions of the initial photon and recoil electron.

From equations (3) and (4) we get

$$pc\cos\theta = h\nu - h\nu'\cos\phi \qquad \rightarrow (5)$$

And $$pc\sin\theta = h\nu'\sin\phi \qquad \rightarrow (6)$$

Thus from (eq. 6)^2+ (eq. 5)^2 we get,

$$p^2c^2 = (h\nu')^2 + (h\nu)^2 - 2h\nu h\nu'\cos\phi \qquad \rightarrow (7)$$

We know, total energy, E = KE + m_0c^2

and relativistic total energy, E = $\sqrt{m_0^2c^4 + p^2c^2}$

Therefore, $(KE + m_0c^2)^2 = m_0^2c^4 + p^2c^2$

or, $p^2c^2 = KE^2 + 2m_0c^2KE \qquad \rightarrow (8)$

Since, KE = $h\nu - h\nu'$, from eq. 8 we get

$$p^2c^2 = (h\nu')^2 + (h\nu)^2 - 2h\nu h\nu' + 2m_0c^2(h\nu - h\nu') \qquad \rightarrow (9)$$

Substituting this value of p^2c^2 in eq. 7 we get,

$$(h\nu)^2 + (h\nu')^2 - 2h\nu h\nu' + 2m_0c^2(h\nu - h\nu') = (h\nu')^2 + (h\nu)^2 - 2h\nu h\nu'\cos\phi$$

or, $2m_0c^2(h\nu - h\nu') = 2h\nu h\nu' - 2h\nu h\nu'\cos\phi \qquad \rightarrow (10)$

or, $m_0c^2(h\nu - h\nu') = h\nu h\nu'(1 - \cos\phi)$

or, $(h\nu - h\nu') = \frac{h\nu h\nu'}{m_0c^2}(1 - \cos\phi) \qquad \rightarrow (10)$

or, $(\nu - \nu') = \frac{h\nu\nu'}{m_0c.c}(1 - \cos\phi)$

or, $\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0c.c}(1 - \cos\phi)$

or, $(\frac{c}{\nu'} - \frac{c}{\nu}) = \frac{h}{m_0c}(1 - \cos\phi)$

$\therefore (\lambda' - \lambda) = \frac{h}{m_0c}(1 - \cos\phi) \qquad \rightarrow (11)$

This was first observed by Arthur H. Compton, that's why this phenomena known as *Compton effect*.

Eq. 11 gives the change in wave length expected for a photon that is scattered through the angle by a particle of rest mass m_0. This change is independent of incident photon wave length, λ.

The quantity $\frac{h}{m_0c} = \lambda_c$, is called the *Compton wavelength* of the scattering particle and thus we can write Eq. 11 as

$(\lambda' - \lambda) = \lambda_c(1 - \cos\phi)$

For $\phi = 180^\circ$, the wave length change will be maximum. For electron the Compton wavelength is $\lambda_c = 2.426 \times 10^{-12}$ m = 2.426 pm. What will be for proton?

The Compton Effect is the chief means by which X-rays lose energy when they pass through matter. During maximum wavelength change electron get maximum kinetic energy.

Wave Properties of Particle

A photon of light of frequency ν has the momentum

$p = \frac{h\nu}{c} = \frac{h}{\lambda}$ ……………….(1)

Since $\nu\lambda = c$, the wavelength of a photon is therefore specified by the momentum according to the relation

$\lambda = \frac{h}{p}$ ……………….(2)

de Broglie suggested that Eq. 2 is applied to material particles as well as to photons. If the momentum of a particle of mass *m* and velocity *v* is $p = mv$, then the de Broglie wavelength is

$\lambda = \frac{h}{mv}$ ……………….(3)

The greater the particle's momentum, the shorter the wavelength is.

Wave particle duality:

We are familiar with the phenomenon of interference, diffraction, polarization, reflection, refraction of light and other com radiation. All these phenomena were explained using the concept of wave nature of light and other electromagnetic radiation.

On the other hand, the black body radiation spectrum, the phenomena of photoelectric effect, and Compton effect strongly demonstrated the particle nature of radiation.

As in the case of electromagnetic waves, the wave and particle aspects of moving bodies can never be observed at the same time, we therefore cannot ask, which the correct description is. All that can be said is that in certain situation a moving body resembles a wave and in other it resembles a particle.

So we accept the dual nature of radiation, when radiation interacts with matter it exhibits its particle character, whereas when radiation interacts with radiation it exhibits the wave character, but never exhibits both the characters simultaneously.

Two examples will help us to appreciate the dual nature:

Find the de Broglie wavelength of (a) 46 *g* golf ball with a velocity of 30 m/s, and electron with a velocity of 10^7 m/s.

- ✓ The wavelength of the golf ball is so small compared with its dimensions that we would not expect to find any wave aspects in its behavior.

- ✓ The dimension of atoms is comparable with this figure the radius of the hydrogen atom for instance, is 5.3×10^{-11} m. It is therefore not surprising that the wave character of moving electrons is the key to understanding atomic structure and behavior.

Problems:

1. Find the shortest wavelength present in the radiation from an x-ray machine where electrons were accelerated by a 50,000 V potential.

✓ **Explanation of λ_{min} in the light of photon**

In order to explain the existence of λ_{min} we must resort to the photon nature of light: The shortest wavelength of the emitted photon gains its energy, $E = hc/\lambda_{min}$, from the maximal loss of the *KE*. of an electron in a single collision. Hence the smallest wavelength of the emitted photon is given by which $K' = 0$ (loss all of its *KE*):

$$\frac{hc}{\lambda_{min}} = K = eV, \text{ or } \lambda_{min} = \frac{hc}{eV} = \frac{6.626\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times5\times10^{4}} = 2.48\times10^{-11} m = 0.0248\, nm$$

(Work function, W_0, is ignored as $W_0 << K$)

Plugging in some typical values, one finds that x-rays lie in the range of 0.01 to 10 nm. Note that the work function W_0 is ignored due to the fact that they are only a few electronvolts, whereas the accelerating potentials that is used to produce x-ray in an x-ray vacuum tube, V, is in the range of 10,000 V. This explains why λ_{min} is the same for different target materials. It is also important to note that the above picture is possible only if we view the x-ray to behave like a particle (i.e. photon). If it were 'wave' the wavelength of the emitted x-ray, according to classical Bremstraahlung, would have been continuous instead of a 'sharp' value, λ_{min}.

2. X-rays of wavelength 10.0 pm are scattered from a target. a) Find the wavelength of the x-rays scattered through 45 degrees. b) Find the maximum wavelength present in the scattered x-rays. c) Find the maximum kinetic energy of the recoil electrons.

3. An X-ray photon of wavelength 0.150 nm collides with an electron at rest and the scattered photon moves off at an angle of 80° from the direction of the incident photon.

   (a) What will be the Compton shift in wavelength?

   (b) What will be the wavelength of the scattered photon?

4. X-rays with a wavelength of 0.12 nm undergo Compton scattering.

   (a) What will be the wavelengths of the photons scattered at angles of 30°, 90° and 150°?

   (b) What will be the energy of the scattered electron for each of these events?

5. What would be the kinetic energy of an electron with a de Broglie wavelength of 5 nm? Through what potential difference should the electron be accelerated to achieve this kinetic energy?

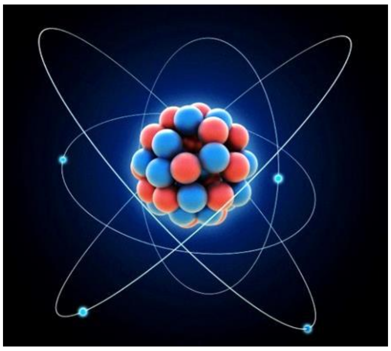
Nuclear physics

The nucleus was discovered in 1911 by Ernest Rutherford in Manchester, England. He and his coworkers took radiation known as alpha particles (see the following) and allowed them to hit a thin gold foil. Although most particles went through or were only slightly deflected, one particle in a thousand was bounced backward from the atoms in the foil. Rutherford compared the experiment to the process of shooting a bullet into a cloud of steam and occasionally finding a bullet bouncing back. The only conclusion that can be drawn in either case is that somewhere inside the atom (or cloud of steam) was a small dense body capable of deflecting fast-moving particles and making them change direction. Rutherford called this small, dense body the nucleus.

Basic properties of Nucleus:

Nuclei have certain time-independent properties such as size, charge, mass, intrinsic angular momentum (nuclear spin) and certain time dependent properties such as radioactive decay and artificial transmutations (Nuclear reactions).

- ✓ Primary constituents of nuclei are protons and neutrons.
- ✓ Nuclear mass is roughly the sum of the masses of its constituents (proton and neutron).
- ✓ Nuclear charge is $+q$ times the number of protons (Z)



The atomic nucleus is designated as $^{A}_{Z}X_{\mathrm{N}}$

Where A → Mass number (N+Z)
N → Neutron number
Z → Atomic number (No. of protons)
X → Chemical symbol of the element.

Atomic masses are measured in atomic mass unit (*amu* or *u*)

$$1\ amu = \frac{1}{12} \times \text{atomic mess of } C^{12} = 1.66054 \times 10^{-27}\ \text{kg} = 931.49\ \text{MeV/c}^2$$

Atomic radius are measured using the following formula

$$R = R_o A^{1/3}$$, Where $R_o = 1.414 \times 10^{-15}$ m is the **atomic radius** of the hydrogen n

Calculation of Nuclear Radii & Ratios

$$R = r_0 A^{\frac{1}{3}} \qquad r_0 = 1.2\times10^{-15}$$

R radius of nucleus A nucleon number

$$\frac{R_1}{R_2} = \frac{A_1^{\frac{1}{3}}}{A_2^{\frac{1}{3}}} \qquad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}}$$

Example 1: What is the diameter of an oxygen nucleus (nucleon number 16)?

$$R_{\mathbf{0}} = r_0 A_{\mathbf{0}}^{\frac{1}{3}} = (1.2\times10^{-15})(16)^{\frac{1}{3}}$$
$$= (1.2\times10^{-15})\times(2.5198)$$
$$= 3.0238\times10^{-15}$$
$$\text{diameter} = 2\times3.0238\times10^{-15} = 6.0476\times10^{-15}$$

Ans. diameter of an oxygen nucleus is 6.05×10^{-15}m

Example 2: If the number of nucleons in a copper nucleus is 64 and the number of nucleons in an oxygen nucleus is 16, how much larger is a copper nucleus than an oxygen nucleus?

$$r_0 = 1.2\times10^{-15}$$

$$\frac{R_{\mathbf{Cu}}}{R_{\mathbf{0}}} = \left(\frac{A_{\mathbf{Cu}}}{A_{\mathbf{0}}}\right)^{\frac{1}{3}} = \left(\frac{64}{16}\right)^{\frac{1}{3}} = (4)^{\frac{1}{3}} = 1.59$$

A copper nucleus is 1.59 times larger than an oxygen nucleus

Constituents of Nucleus:

Proton → 1.6726×10^{-27} kg = 1.007176 u = 938.28 MeV/c^2

Neutron →1.6750×10^{-27} kg = 1.008665 u = 939.57 MeV/c^2

${}^{1}_{1}H$ atom → 1.6736×10^{-27} kg = 1.007825 u = 938.79 MeV/c^2

Electron → 9.1095×10^{-31} kg = 5.486×10^{-4} u = 0.511 MeV/c^2

Types of Nuclei:

Nuclide → A specific nuclear species

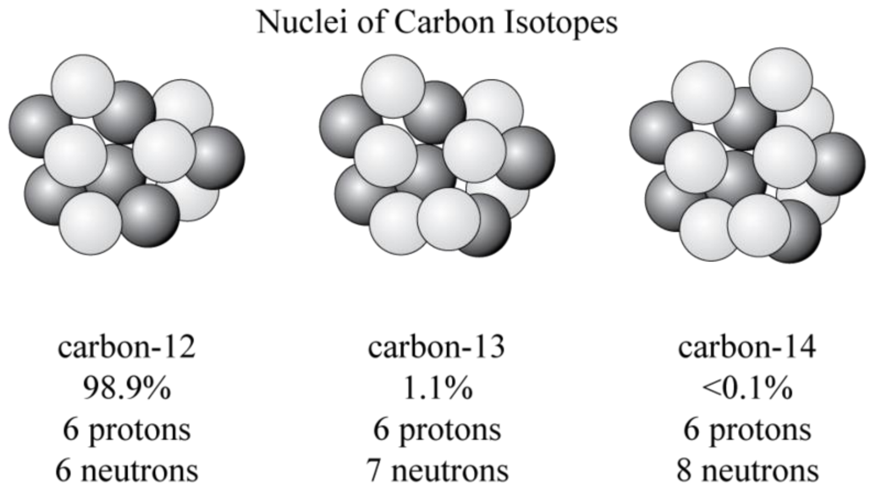
Nucleon → Neutron or proton

Isotopes → Nuclides of same Z and different N. Example: (${}^{16}_{8}O_8$, ${}^{16}_{8}O_9$, ${}^{16}_{8}O_{10}$)

Isotones → Nuclides of same N and different Z. Example: ($^{14}_{7}N_{7}$, $^{15}_{8}O_{7}$, $^{16}_{9}F_{7}$)

Isobars → Nuclides of same atomic mass no (A). Example: ($^{14}_{6}F_{8}$, $^{14}_{7}N_{7}$, $^{14}_{8}O_{6}$,)

Isomers → Nuclide in an excited state with measurable half life. $^{16}_{8}O_{8}$*



Why electron cannot be in the nucleus?

- **Nuclear Size:** To confine an electron in a box of nuclear dimension, it must have energy more than 20 MeV. Whereas, electrons emitted during β^- decay have energies of only 2 or 3 MeV. On the other case for proton this box confinement energy is only 0.2 MeV.
- **Nuclear Spin**: Deuterium if consists two protons and an electron, its nuclear spin should be $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{2}{3}$ or, $\frac{1}{2} + \frac{1}{2} - \frac{1}{2} = \frac{1}{2}$. But it is actually 1.
- **Magnetic moment**: Magnetic moment of proton = 0.15% of electron. If electron is part of nucleus its magnetic moment ought to be of the order of magnitude of the electron. But it is only comparable to that of proton.
- **Electron- nuclear Interaction**: BE 8 MeV per particle for nucleon. If electron is inside the nucleus how can the other electrons is the atom remain outside?

Binding Energy:

Mass of H nucleus	= 1.007825 u
Mass of Neutron	= 1.008665 u
Expected mass of	= 2.016490 u

But mass of the atom is 2.014102 u. Thus (2.016490 – 2.014102) u or 0.002388 u mass is missing (→ Mass defect.)

Now, the energy needed to work up a deuterium nucleus into separate neutron and proton is 2.224 MeV. Thus Binding Energy can be defined as -

The energy equivalent to mass difference between the sum of the masses of individual nucleons (protons and neutrons) and the mass of the nucleus. That is the energy equivalent to the missing mass of a nucleus. It represents the energy needed to dissociate the nucleus into separate nucleons. The more the B.E. the more energy that must be supplied to break up the nucleus.

Therefore the binding energy E_b in of a Nucleus can be written as

$$E_B = [\, Z\, m_p + N m_n - m(A,Z)\,]\, c^2$$

or, $$E_B = [\, Z\, m_p + N m_n - m(A,Z)\,]\; 931.49 \text{ Mev/u}$$
$$= (\Delta m)c^2$$

Now average binding energy per nucleon

$$E_{B,avg}\,(A,Z) = \frac{E_{B\,Total}}{A},$$

Thus, binding energy per nucleon is the energy that is needed to separate an proton or neutron form the nucleus. This is also called separation energy.

Problem: BE of the Neon isotope ${}^{20}_{10}Ne$ is 160.647 MeV. What is its atomic mass?

Sol^n : Here Z = 10, N= 10, m_p = 1.007825 u, m_n = 1.008665 u.

$$\therefore m(A,Z) = [(Z\, m_p + N m_n) - (E_B/931.49)]$$
$$= [(10\times 1.007825 + 10\times 1.008665) - (160.647/931.49)]$$
$$= 19.992 \text{ u}$$

B. E Curve: If we plot B. E per nucleon $E_{B,avg}$ in MeV as a function of mass number we get the curve rises sharply at first and then rises gradually until it reaches a maximum of 8.79 MeV at A= 56, Corresponding to ${}^{56}_{26}Fe$ and thereafter, drops slowly to about 7.6 MeV at the highest mass numbers. Thus, the nuclei with intermediate mass range are more stable and if higher mass number nuclei split into lighter ones or light nuclei joined together some energy will be released. The peaks in the curve show the existence of isobars.