Kinetic Theory of Gases

Equation of state:
The state of a certain mass m of a material is described by its pressure P, volume V, and temperature T. There is a functional relationship among these quantities, which is

$$V = f(P, T, m)$$

That is, the volume of a certain mass of material depends on the pressure, temperature, and mass. Any such relationship is called an equation of state.

Vapor pressure:
The pressure of vapor in equilibrium with liquid or solid at any temperature is called vapor pressure of the substance at that temperature. The vapor pressure of a substance is a function of temperature only, not of volume.

Humidity:
Atmospheric air is a mixture of gases consisting of 78% nitrogen, 21% oxygen and small amount of carbon-dioxide, water vapor, etc. The mass of water vapor per unit volume is called absolute humidity.

Partial pressure:
The total pressure exerted by the atmosphere is the sum of the pressure exerted by the component gases; these pressures are called partial pressure.

Molecular theory of matter:
Matter in all phases is composed of particles called molecules. The intermolecular force which hold molecules of solid and liquid together are mainly electrical in nature. When the molecules are far apart, as in gas, the intermolecular force is very small and attractive. When the gas is compressed, the force increases. In liquids, relatively large pressure is required to compress the substance appreciably.

Avogadro's number:
One mole of any pure chemical contains a definite number of molecules, the same number for all compounds. The number of molecules in a mole is called Avogadro's number and is denoted by N_A.

$$N_A = 6.023 \times 10^{23} \text{ molecules mol}^{-1}$$

If m is the mass of a single molecule and M is the molecular mass of the compound, then the mass of one mole is

$$M = N_A m$$

For example, the mass of one mole of hydrogen molecule is 2gm (molecular mass). It means that 2gm of hydrogen contains 6.023×10^{23} molecules.

Equation of state for an ideal gas:
At low densities, all gases, no matter what their chemical composition, tend to show a certain relationship among the thermodynamic variables; the pressure P, volume V and temperature T. This suggests the concept of an ideal gas, one that would have the same simple behavior under all conditions.
We can find the relationship by considering a gas of pressure P, volume V, and temperature T. We find that

a) For a given mass of gas held at a constant temperature the pressure is inversely proportional to the volume, i.e.

$$V \propto \frac{1}{P} \text{ (At constant } T\text{)}$$

This is known as Boyle's law.

b) For a given mass of gas held at a constant pressure the volume is directly proportional to the temperature, i.e.

$$V \propto T \text{ (At constant } P\text{)}$$

This is known as Charles law.

c) For a given mass of gas held at a constant volume the pressure is directly proportional to the temperature, i.e.

$$P \propto T \text{ (At constant } V\text{)}$$

This is known as pressure law.

We can summarize these three experimental results by the relation

$$\frac{PV}{T} = \text{constant}$$

$$\frac{PV}{T} = nR$$

$$\therefore PV = nRT$$

where n is the number of moles and R is known as the universal gas constant having a value $R = 8.314$ $Joule\ mole^{-1}\ {}^{o}K^{-1}$.

Therefore, we define an ideal gas that obeys this relation under all conditions.

Assumption of an ideal gas:

From the microscopic point of view we can define an ideal gas by making following assumptions.

1. A gas consists of particles called molecules.
2. The molecules are in random motion and obey Newton's laws of motion.
3. The total number of molecules is large.
4. The volume of molecules is a negligibly small fraction of volume occupied by the gas.
5. No appreciable forces act on the molecules except during a collision.
6. Collisions are elastic and are of negligible duration.

Exercise 17-3: A tank consists of $0.5m^3$ of nitrogen at $27^{\circ}C$ and $1.5\times10^5 Pa$. What is the pressure if the volume is increased to $5.0m^3$ and the temperature is increased to $327\ ^{\circ}C$?

Solution:

Given that

$$V_1 = 0.5m^3, T_1 = 27^oC = 300K, P_1 = 1.5\times10^5 Pa, V_2 = 5m^3, T_2 = 327^oC = 600K$$

We know

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow P_2 = \frac{P_1V_1T_2}{T_1V_2} = \frac{1.5\times10^5\times0.5\times600}{300\times5}$$

$$\therefore P_2 = 3\times10^4 Pa$$

Exercise 17-4: A tank having a volume of $0.10m^3$ is filled with oxygen at a gauge pressure of $4.0\times10^5 Pa$ and temperature of 47^oC. Later it is found that, because of a leak, the gauge pressure has dropped to $3.0\times10^5 Pa$ and temperature has decreased to 27^oC. Find a) the initial mass of oxygen and b) the mass that has leaked out.

Solution:
Given that

$$V_1 = 0.1m^3, T_1 = 49^o C = 320K, P_1 = 1.01\times10^5 + 4\times10^5 = 5.01\times10^5 Pa,$$

$$P_2 = 1.01\times10^5 + 3\times10^5 = 4.01\times10^5 Pa, V_2 = 0.1m^3, T_2 = 27^o C = 300K$$

We know

$$PV = nRT$$

a) Therefore

$$n = \frac{P_1V_1}{RT_1} = 18.8 moles$$

Total mass = no. of moles × molar mass of oxygen = $18.8 \times 32 = 602gm = 0.602kg$.
b) Again

$$\therefore n = \frac{P_2V_2}{RT_2} = 16.08 moles$$

Therefore
Mass of the gas remaining in the tank = $16.08 \times 32 = 514.72gm$
Mass of the gas leaked out = $602 - 514.72 = 87.2gm$

Problems for practice: Exercise 17- 5, 17- 12.