Special Theory of Relativity

Introduction:

Galileo was first persuaded (coaxed) that earth is in motion around the sun, which is stationary. Most of his fellows (contemporaries) argued that if it is true then why birds leaving the earth would not be left behind by the speeding earth? Galileo reasoned that if a ship moved uniformly on the sea, a sailor could not distinguish between the situations either the ship is in motion and the sea is at rest or the ship is at rest and the sea is in motion. In Galileo's view- the only motion that is measurable is the relative motion between the ship and the sea; hence the term "Relativity" was manifested first ever.

Relative Motion:

The first step is to clarify what we mean by motion. Using Cartesian co-ordinate system in three dimensions, we can characterize any location by three co-ordinates of that space point. When we say that something is moving, what we mean is that its position relative to something else is changing. That is all motion is relative. In each case a frame of reference is part of the description of the motion.

Frame of Reference:

In order to describe the motion of moving bodies, we need to state where the object is at any given time. But to state where an object is, we need to measure its position relative to something else, right? So we need a reference point from which to define the position of objects. Once we have chosen such a point, which is called the origin, we can specify the position of the object by saying, for instance, that the object is distance x to the east, distance y to the north, and distance z up from the origin. We also need a clock so that we can specify at what time t the object was at the given position.

The co-ordinate system, relative to which any measurement can be done to specify the state of rest or motion of an object, is known as the frame of reference.

When we have the origin and the directions in which to measure the distance from the origin set up, and a clock to measure the time, we say that we have a frame of reference or simply a frame.

Inertial and Non-inertial Frame of reference:

Newton's first law, the law of inertia, states that

1. if an object is at rest it will stay at rest if no force is acting on it, and
2. if an object is moving it will keep on moving at constant velocity if no force is acting on it.

This law is actually not always correct! (Surprised?) It depends on which frame you are using to describe the motion of the object. For instance, if you are using the moving object itself as the origin of your frame of reference, it is always at rest no matter what forces are acting on it.

So when we talk about the law of inertia, we are assuming that a frame exists in which the law is correct. Such a frame is called an inertial frame. If one such inertial frame exists, then an infinite number of other inertial frames exist since any frame that is moving at a constant relative velocity to the first inertial frame is also an inertial frame.

Inertial Frame:

An inertial frame of reference is one in which Newton's first law of motion holds good. In this frame, an object at rest remains at rest and an object in motion continues to move at constant velocity (constant speed and direction) if no force acts on it. Any frame of reference that moves at constant velocity relative to an inertial frame is itself an inertial frame.

Special theory of relativity deals with problems that involve inertial frames of reference that is, where the body moves with a constant velocity.

Non-inertial Frame:

A reference frame in which a body is accelerated without being acted upon by external force is called a non-inertial frame of reference. Newton's laws are not valid in such a frame of reference.

General theory of relativity, which is published by Einstein a decade later in 1917, concerns itself with all frames of reference including the non-inertial frames of reference which are accelerated with respect to one another.

Failure of Newtonian Mechanics:

For an electron accelerated through a 10 MEV potential difference, a value reasonable easy to obtain, the speed u equals 0.9988c.

If the energy of 10 MeV electron above is increased by a factor of four (to 40 MeV) experiment shows that the speed is not doubled to 1.9976c as we might expect from the Newtonian relation $K = \frac{1}{2}Mv^2$, but remain below c; it increases only from 0.9988c to 0.9999c, a chance of 0.11 percent.

Galilean Transformation:

Why Galilean transformation introduced?

Newton's laws of motion does not tell us whether there is one or many inertial frames of reference, nor, if there is more than one, does it tell us how we are to relate the coordinates of an event as observed from the point-of-view of one inertial reference frame to the coordinates of the same event as observed in some other. In establishing the latter, we can show that there is in fact an infinite number of inertial reference frames. Moreover, the transformation equations that we derive are then the mathematical basis on which it can be shown that Newton's Laws are consistent with the principle of relativity. The transformation of co-ordinates of a particle from one inertial frame of reference to another is called the Galilean transformation.

Let us consider we are in an inertial frame of reference S and the coordinates of some event that occurs at the time t are (x, y, z). An observer located in a different inertial frame S′ which is moving with respect to S at constant velocity **u**, will find that the same event occurs at the time t′ and has the coordinates (x′, y′, z′). For convenience, let **u** is in the +x direction as shown in the figure.

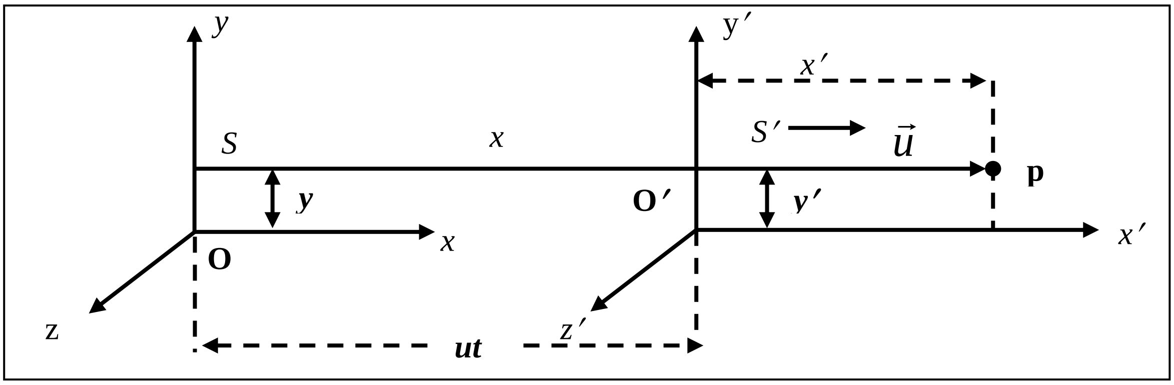


Fig. 6: Frame S′ moves in the X-direction with speed u relative to frame S.

Let us suppose that the clocks in S and S′ are set such that when the origins of the two reference frames O and O′ coincide, all the clocks in both frames of reference read zero i.e. t = t′ = 0. According to 'common sense', if the clocks in S and S′ are synchronized at t = t′ = 0, then they will always read the same, i.e. t = t′ always. This is the absolute time concept. We consider an event of some kind, i.e. an explosion occurs at a point (x′, y′, z′ , t′) according to S′. The motion is in the +x direction and there is no relative motion in the y and z directions, and so the event occurs in S′ at the point

$$\left.\begin{aligned} x' &= x - ut \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\} \qquad (1)$$

These equations together are known as the Galilean transformation. They tell us how the coordinates of an event in the inertial frame S′, which moving with a uniform velocity **u** with respect to S, are related to the coordinates of the same event as measured in S which is in rest.

According to the symmetry of space, if S′ is moving with a velocity **u** with respect to S, then S will be moving with a velocity **-u** with respect to S′ so the inverse transformation should be obtainable by simply exchanging the primed and unprimed variables, and replacing u by −u. Thus the inverse transformation, i.e., the transformation from S′ to S is

$$\left.\begin{aligned} x &= x' + ut' \\ y &= y' \\ z &= z' \\ t &= t' \end{aligned}\right\} \qquad (2)$$

The time-interval and space-interval measurements are absolutes according to the Galilean transformation.

Special relativity is based on two postulates which are contradictory in classical mechanics:

It was Einstein who, in 1905, pointed out that the only way to understand this was to change our notion of simultaneity. This was his famous special theory of relativity.

1. Principle of equivalence of physical laws: “The laws of physics are the same in all inertial frame of reference. No preferred inertial system exists”.
2. Principle of constancy of velocity of light: “The speed of light in free space (vacuum) has the same value $c \cong 3\times10^8$ m/s in all inertial frames of reference, independent of the relative velocity of the source and observer”.

Consequences of Einstein’s postulates:

There are four important consequences of Einstein’s postulates:

Relativity of simultaneity: Two events that appear simultaneous to an observer A will not be simultaneous to an observer B if B is moving with respect to A.

Relativity of Time (Time dilation): Moving clocks tick slower than an observer's "stationary" clock.

Relativity of Length (Length contraction): Objects are observed to be shortened in the direction that they are moving with respect to the observer.

Mass-energy equivalence: According to the relationship $E = mc^2$, energy and mass are equivalent and transmutable.

Failure of Galilean transformation:

The second postulate calls for the same value of the speed of light c when determined in S or S′. This contradicts with Galilean transformation. If we measure the speed of light in the x-direction in the S system to be c, however, in the S′ system it will be

$$c' = c - u$$

Clearly a different transformation is required if the postulates of special relativity are to be satisfied.

Lorentz Transformation:

Einstein's special theory of relativity says the speed of light is constant; we have to modify the way in which we translate the observation in one inertial frame to that of another. The set of Galilean transformation

$$x' = x - ut\,,\ t' = t \qquad (1)$$

is wrong and are not consistent with the experimental results. The correct relations which are consistent with the experimental results are

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}} \qquad t' = \frac{t - \frac{u}{c^2}x}{\sqrt{1 - u^2/c^2}} \qquad (2)$$

These equations are called the Lorentz transformation.

Derivation of Lorentz Transformation Equations:

For the constancy of velocity of light we have to introduce the new transformation equations which fulfill the following requirements:-

1. The speed of c must have the same value in every inertial frame of reference.
2. The transformations must be linear and for low speeds u<<c, they should approach the Galilean transformations.
3. They should not be based on "absolute time" and "absolute space".

A reasonable guess about the nature of the correct relationship between x and x′ is

$$x' = k(x - ut) \qquad (1)$$

Here k is a factor that does not depend on either x or t but may be a function of u.

Because the equations must have the same form in both S and S′, we need only change the sign of u (in order to take into account the difference in the direction of relative motion) to write the corresponding equation for x in terms of x′ and t′:

$$x = k(x' + ut') \qquad (2)$$

The factor k must be the same in both frames of reference since there is no difference between S and S′ other than in the sign of u.

Let us consider that a light pulse that starts at the origin of S at t = 0. Since we have assumed that the origins are coincident at t′ = t = 0, the pulse also starts at the origin of S′ at t′ = 0. Einstein's postulates require that the equation for the x-component of the wave front of the light pulse is

$x = ct$ in frame S and $x' = ct'$ in frame S′.

Substituting ct for x and ct′ for x′ in Eq. (1) and (2), we get

$$ct = k(ct' + ut') = k(c+u)t' \qquad (3)$$

and

$$ct' = k(ct - ut) = k(c-u)t \qquad (4)$$

From equation (3), $\frac{t'}{t} = \frac{c}{k(c+u)}$

Similarly from equation (4), $\frac{t'}{t} = \frac{k(c-u)}{c}$ and thus we get

$$\frac{c}{k(c+u)} = \frac{k(c-u)}{c} \Rightarrow k^2 = \frac{c^2}{c^2 - u^2}$$

$$k^2 = \frac{1}{1 - u^2/c^2} \Rightarrow k = \frac{1}{\sqrt{1 - u^2/c^2}} \qquad (5)$$

Substituting $x' = k(x - ut)$ for x′ in $x = k(x' + ut')$, we get

$$x = k[k(x - ut) + ut']$$
$$= k^2(x - ut) + kut'$$

$$\text{or } t' = \frac{x - k^2(x-ut)}{ku}$$

$$= \frac{x - \dfrac{(x-ut)}{(1-u^2/c^2)}}{u} \times \sqrt{1-u^2/c^2}$$

$$= \frac{x - u^2/c^2 x - x + ut}{u\sqrt{1-u^2/c^2}} = \frac{ut - u^2/c^2 x}{u\sqrt{1-u^2/c^2}}$$

$$= \frac{t - \dfrac{u}{c^2}x}{\sqrt{1-u^2/c^2}}$$

$$t' = \frac{t - \dfrac{u}{c^2}x}{\sqrt{1-u^2/c^2}} = k(t - \frac{u}{c^2}x) \qquad (6)$$

The complete relativistic transformation is

$$x' = k(x-ut) \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad (7)$$

$$y' = y \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad (8)$$

$$z' = z \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad (9)$$

$$t' = k(t - \frac{ux}{c^2}) \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad (10)$$

These are the Lorentz transformation equations.

(i) Lorentz transformation equation is linear in x and t.

(ii) Reduces to Galilean transformation for u/c<<1.

The observer in S′ will observe that the frame S is moving to the right with a velocity –u with respect to it. Thus, when we solve Eqs. (7)-(10) for x, y, z, and t in terms of the primed coordinates, we obtain

$$x = k(x' + ut') \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad (11)$$

$$y = y' \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad (12)$$

$$z = z' \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad (13)$$

$$t = k(t' + \frac{ux'}{c^2}) \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad (14)$$

which are identical in form of Eqs. 7-10 and are known as Inverse Lorentz transformations.

Relativistic Velocity Transformation:

The complete set of relativistic Lorentz transformation is

$$x' = k(x-ut), \qquad y' = y, \qquad z' = z \qquad \text{and} \qquad t' = k(t - \frac{ux}{c^2}) \ldots\ldots \quad \ldots\ldots (1)$$

Let v_x, v_y, and v_z are the velocity components of a particle with respect to the S frame along x, y and z directions respectively, while v_x', v_y', and v_z' are the velocity components of that particle with respect to S′ frame along x′, y′, and z′ directions respectively.

By differentiating the Lorentz transformation equations for x′, y′, z′ and t′, we get

$$dx' = k(dx - udt), \qquad dy' = dy, \qquad dz' = dz \text{ and } \quad dt' = k(dt - \frac{u}{c^2}dx)$$

Therefore, $$v'_x = \frac{dx'}{dt'} = \frac{k(dx - udt)}{k(dt - \frac{u}{c^2}dx)} = \frac{\frac{dx}{dt} - u}{1 - \frac{u}{c^2}\frac{dx}{dt}} = \frac{v_x - u}{1 - \frac{u}{c^2}v_x} \qquad (5)$$

We may write the complete set of relativistic velocity transformation as

$$v'_x = \frac{v_x - u}{1 - \frac{u}{c^2}v_x}, \qquad v'_y = \frac{v_y\sqrt{1-u^2/c^2}}{1 - \frac{u}{c^2}v_x} \; and, \qquad v'_z = \frac{v_z\sqrt{1-u^2/c^2}}{1 - \frac{u}{c^2}v_x}$$

The inverse velocity transformation equations are

$$\boldsymbol{v_x = \frac{v'_x + u}{1 + \frac{u}{c^2}v'_x}, \qquad v_y = \frac{v'_y\sqrt{1-u^2/c^2}}{1 + \frac{u}{c^2}v'_x} \; and, \qquad v_z = \frac{v'_z\sqrt{1-u^2/c^2}}{1 + \frac{u}{c^2}v'_x}}$$

(i) If u and v_x are very small compared to c, the velocity transformations satisfy the classical results.

$$v'_x = v_x - u, \qquad v'_y = v_y \; and, \qquad v'_z = v_z$$

(ii) If a ray of light is emitted in the moving frame S' in its direction of motion relative to S, then $v'_x = c$, and no matter what the value of u of the observer, an observer in frame S will measure the speed

$$v_x = \frac{v'_x + u}{1 + \frac{u}{c^2}v'_x} = \frac{c + u}{1 + \frac{u}{c^2}c} = c$$

Thus the observers in the car and on the road both find the same value for the speed of light, as they must.

Example #1: Two electrons leave a radioactive sample in opposite directions, each having a speed of $0.67c$ with respect to the sample. What is the speed of one electron relative to the other?

Solution: Here $v_{x'} = 0.67c$ and u = 0.67c

The speed of one electron relative to other $v_x = \frac{v'_x + u}{1 + \frac{u}{c^2}v'_x} = \frac{0.67c + 0.67c}{1 + \frac{0.67c}{c^2} \times 0.67c} = \frac{1.34c}{1.45} = 0.92c$

This is means that speed of one electron relative to the other is less than c.

Relativity of Simultaneity:

Two events are said to be simultaneous if they occur at the same time. According to the relativity of simultaneity, if two observers are in relative motion, they will not agree as to whether two events are simultaneous. If one observer finds them to be simultaneous, the other generally will not, and conversely.

The simultaneity of the two events is not an absolute concept and depends on the frame of reference. In fact each observer is correct in his own frame of reference.

Two events that appear simultaneous to an observer A will not be simultaneous to an observer B if B is moving with respect to A.

Let us consider an example to clarify the above statement:

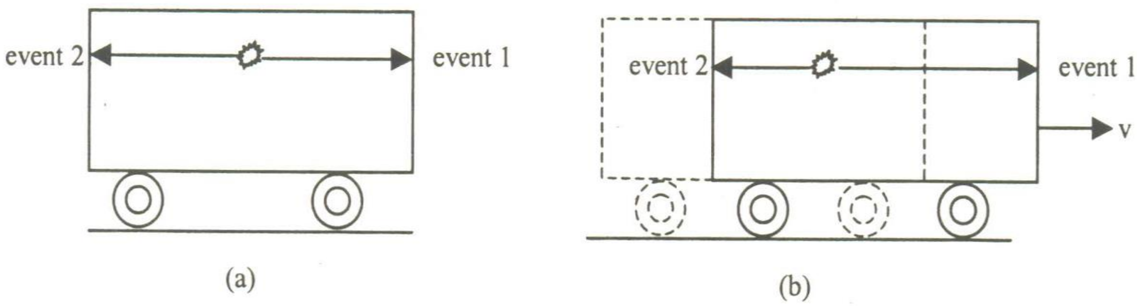


Figure: Spreading of light signals in a toy car as observed (a) by an observer on the car itself and (b) by an observer standing on the ground.

Imagine a trolley traveling at a constant speed along a smooth, straight track. In the centre of the trolley there a light bulb is hanged. When it is switched on, the light spreads out in all directions at a speed c. Because the lamp is equidistant from the two ends, an observer on the trolley will find that the light reaches the front and the rear ends at the same time, i.e., the two events of light reaching the front and the rear ends occur simultaneously (Fig. a). However, to an observer on ground these two events do not appear to be simultaneous. As the light travels out from the bulb, the trolley itself moves forward, so the beam going to the rear end has a shorter distance to travel than the one going forward. According to this observer, therefore, the second event appears to happen before the first event (Fig. b). Therefore, we can conclude that the two events that are simultaneous in one inertial frame are not, in general, simultaneous in another frame.

In fact, measuring times and time intervals involve the concept of simultaneity and from the above discussion it follows that the time interval between two events may be different in different frames of references.

Proof of this statement:

Let us consider two frames of reference S and S′ . The frame reference S′ is moving with velocity u relative to the frame of reference S along +ve direction of x-axis and the two events occur simultaneously in S. Since the events are simultaneous in frame S, therefore we have $t_1 = t_2$. If t_1' and t_2' are the corresponding times of the same two events with respect to system S′, then we have from Lorentz transformation equations:- $t_1' = k(t_1 - \frac{ux_1}{c^2})$ and $t_2' = k(t_2 - \frac{ux_2}{c^2})$

$$\therefore \quad \begin{aligned} t_2' - t_1' &= k(t_2 - \frac{ux_2}{c^2}) - k(t_1 - \frac{ux_1}{c^2}) \\ &= k\frac{u(x_1 - x_2)}{c^2} \quad \text{since} \quad t_1 = t_2 \end{aligned}$$

Thus if the events are simultaneous in frame S′, t_1' must be equal to t_2' or $t_2' - t_1'$ must be equal to zero, but it is not so because x_1 is not equal x_2. Therefore, the same two events are not simultaneous in frame S′.

Length Contraction:

The length of a body is measured to be greatest when it is rest relative to the observer. When it moves with a velocity v relative to the observer its measured length is contracted. In simple terms: The length of an object to be shorter when it is moving then when it is rest.

Situation : A stick fixed in S′ but observed in S.

Let us consider a stick at rest in the system S′ and the coordinates of two ends are x'_1 and x'_2 so that its length as measured by an observer in S′ is given by $L' = x'_2 - x'_1$. The frame S′ is moving with a velocity u with respect to S.

From S, length must be measured at the same time t. According to the Lorentz transformation $x_1' = k(x_1 - ut)$ and $x_2' = k(x_2 - ut)$.

Hence $L' = (x_2' - x_1') = k(x_2 - x_1) = \dfrac{L}{\sqrt{1 - u^2/c^2}} \qquad \Rightarrow L = L'\sqrt{1 - u^2/c^2}$

This summarizes the effect known as length contraction. The frame S′ is at rest with respect to the object so the measured rest length is L′ and L′ >L. As the frame S′ is moving with respect to S, all observers outside the frame S′ are in motion relative to S′ and measure a shorter length, **but only along the direction of motion**; length measurements transverse to the direction of the motion are unaffected.

For ordinary speeds ($u \ll c$), the effects of length contraction are too small to be compared ($L' \approx L$).

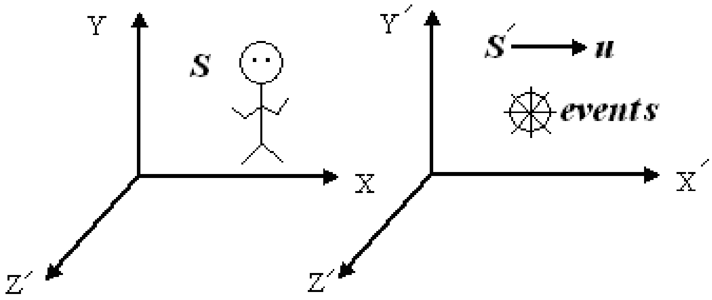
Faster means shorter

Length contraction suggests that objects in motion are measured to have a shorter length than they do at rest.

Time Dilation:

Moving clocks tick slower than an observer's "stationary" clock.

A clock is measured to go at its fastest rate when it is at rest relative to the observer. When it moves with a velocity u relative to the observer, its rate measured to have slowed down by a factor $\sqrt{1-u^2/c^2}$.



Proof: Let there be two frames of reference S and S′ ; S′ is moving with a velocity u relative to S along (+ve) direction of x-axis. Let a clock to be at rest at the point x′ in the moving frame S′ and another clock be rest at the point x in the frame S.

Let two events occurs in S′ at x′ : one occurs at time t'_1 and other at time t'_2. The corresponding times measured by the clock at S are t_1 and t_2 respectively.

Then the time interval between the two events as noted on the clock in the moving frame S′ is given by

$\Delta t' = t'_2 - t'_1$ and the time interval between the same two events as noted by the clock in the stationary observer S is given by $\Delta t = t_2 - t_1$.

According to the Inverse Lorentz transformation $t_1 = k(t'_1 + ux'/c^2)$ and $t_2 = k(t'_2 + ux'/c^2)$

Therefore, from these equations, we have $\Delta t = t_2 - t_1 = k(t'_2 - t'_1) = \dfrac{\Delta t'}{\sqrt{1-u^2/c^2}}$

Now since $\dfrac{1}{\sqrt{1-u^2/c^2}}$ is greater than unity as u/c <1, therefore $\Delta t > \Delta t'$.

Thus the time interval $\Delta t'$ between two events occurring at a given point in the moving frame S′ appears to be longer or dilated to the observer in the stationary frame S.

The relation is true only when $\Delta t'$ represents the time interval between two events in a reference frame where the two events occur at the same point in space.

The time dilation effect has been verified experimentally with decaying elementary particles as well as with precise atomic clocks carried aboard aircraft.

Meson Decay:

μ-meson Produced in the upper reach of the atmosphere as a result of collision between fast cosmic ray particles, arriving the earth from space, and the air molecules.

It decays into electron and two neutrinos each.

μ-meson mass = 215 × Mass of an electron

Lifetime = 2.2 × 10^{-6} sec

Velocity = 0.998c

Travel Distance = 660 m

Observed Travel Distance = 10 km

Calculated Travel Distance = 10479 m (Using Length Contraction)

Observed Lifetime = 34.92 × 10^{-6} sec

Calculated Travel Distance = 10455 m (Using Time Dilation)

Einstein's Mass And Energy Relation:

Let a force F is acting on a body so that there is a displacement ds along the direction of the force. The work done is then given by W = Fds

If no other forces act on the object and the object starts from rest, all the work done on it becomes kinetic energy K. and is given by $K = \int_0^s Fds$ (1)

If the force is changing with time and is given by $F = \frac{dp}{dt} = \frac{d(mv)}{dt}$. Where v is the velocity of the body and the relativistic momentum is $p = mv = \frac{m_0 v}{\sqrt{1-v^2/c^2}}$.

So the kinetic energy K becomes

$$K = \int_0^S Fds = \int_0^S \frac{d(mv)}{dt} ds = \int_0^v d(mv)\frac{ds}{dt}$$

$$= \int_0^v (mdv + vdm)v$$

$$= \int_0^v (mvdv + v^2 dm) \qquad (2)$$

in which m and v are variable. These quantities are related as

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}} \Rightarrow m^2(1-v^2/c^2) = m_0 \Rightarrow m^2c^2 - m^2v^2 = m_0{}^2c^2 \qquad (3)$$

Differentiating equation (3), we get

$$2mc^2dm - m^2 2vdv - v^2 2mdm = 0$$

$$\Rightarrow mvdv + v^2dm = c^2dm \qquad (4)$$

Substituting this value in equation (2) and integrating, we get

$$K = \int_0^v c^2 dm$$

$$= c^2 \int_{m=m_0}^{m=m} dm$$

$$K = \int_0^v c^2 dm$$

$$= c^2 \int_{m=m_0}^{m=m} dm$$

$$= mc^2 - m_0c^2 \qquad (3)$$

This is the relativistic equation for kinetic energy of a body moving with a velocity v. Also, if we take mc^2 =E as the total energy of the body then the above equation may be written as

$$\mathrm{E} = m_0c^2 + K \qquad (4)$$

in which m_0c^2 is called the rest energy of the body.

The rest energy is energy of the body at rest when v = 0, and K = 0 and the body has an amount of

$$\text{Rest energy } E_0 = m_0c^2 \qquad (5)$$

Thus we can write $E = E_0 + K$ (6)

If the body is moving, its total energy is $E = mc^2 = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}}$ (7)

This is the Einstein famous mass-energy relation.

(i) For low speeds, $v/c << 1$, Then we can write

$$K = mc^2 - m_0 c^2 = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}} - m_0 c^2 = m_0 c^2 \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1 \right) \quad (8)$$

Using the binomial approximation $(1+x)^n \approx 1 + nx$ with $|x| << 1$, we get

$$\begin{aligned} K &= m_0 c^2 \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1 \right) \\ &= m_0 c^2 \left(1 + \frac{1}{2} \frac{v^2}{c^2} - 1 \right) \\ &= \frac{1}{2} m_0 v^2 \end{aligned} \quad (9)$$

At low speed, the relativistic expression for the K.E. of a moving body reduces to the classical one.

(ii) Mass-less particles: We know, total energy $E = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}}$, (10)

and relativistic momentum $p = \frac{m_0 u}{\sqrt{1-v^2/c^2}}$ (11)

When $m_0 = 0$ and v<c, i.e. E = p = 0 i.e. **A mass less particle with a speed less than that of light can have neither energy nor momentum.**

(iii) When m_o =0 and v = c, E = 0/0 and p = 0/0 which are indeterminate: E and p can have any values. The equations [10] & [11] are consistent with the existence of mass-less particles that possess energy and momentum provided that they travel with the speed of light.

(iv) There is another restriction on mass-less particles.

we can write $E^2 = \frac{{m_0}^2 c^4}{1-v^2/c^2}$

Also we can write $p^2 = \frac{{m_0}^2 v^2}{1-v^2/c^2}$, or, $p^2 c^2 = \frac{{m_0}^2 v^2 c^2}{1-v^2/c^2}$

Subtracting $p^2 c^2$ from E^2 yields

$$E^2 - p^2c^2 = \frac{m_0{}^2c^4}{1-v^2/c^2} - \frac{m_0{}^2v^2c^2}{1-v^2/c^2}$$

$$= m_0{}^2c^2\left(\frac{c^2}{c^2-v^2} - \frac{v^2}{c^2-v^2}\right)c^2$$

$$= m_0{}^2c^4\left(\frac{c^2-v^2}{c^2-v^2}\right)$$

$$= m_0{}^2c^4 \qquad (12)$$

$\therefore \qquad E^2 = m_0{}^2c^4 + p^2c^2$

Therefore, for all particles, we have

$$E = \sqrt{m_o{}^2\,c^4 + p^2c^2} = \sqrt{E_o{}^2 + p^2c^2} \qquad (13)$$

According to this formula, if a particle exists with $m_0 = 0$, the relationship between its energy and momentum must be given by

$$\text{mass - less particles} \qquad E = pc \qquad (14)$$

In fact, mass-less particles of two different kinds - the photon and the neutrino have indeed been discovered and their behavior is as expected.

1. **Find the mass of an electron (m_o =9.1x10^{-31} kg) whose velocity is 0.99c.**
   Solution:
   Given u/c=0.99, so $m = \dfrac{m_0}{\sqrt{1-u^2/c^2}} = \dfrac{9.1x10^{-31}}{\sqrt{1-(0.99)^2}} = 64x10^{-31}kg.$
   which is 7 times greater than the electron's rest mass. If v =c , m $=\infty$, from which we can conclude that v can never equal c:

No material object can travel as fast as light

2. **A stationary body explodes into two fragments each of rest mass 1.0 Kg that move apart at speeds 0.6C relative to the original body. Find the rest mass of the original body.**
   Solution:
   The total energy of the original body must equal the sum of the total energies of the fragments. Hence
   $E = E_0 + K.E = E_0 + 0 = E_0$
   $$m_0c^2 = \frac{m_{01}c^2}{\sqrt{1-u_1{}^2/c^2}} + \frac{m_{02}c^2}{\sqrt{1-u_2{}^2/c^2}} = \frac{(1.0kg)c^2}{\sqrt{1-(0.6)^2}} + \frac{(1.0kg)c^2}{\sqrt{1-(0.6)^2}}$$
   Therefore,
   $m_0 = 2.5kg$
   Mass can be created or destroyed but when this happens an equivalent amount of energy simultaneously vanishes or comes into being and vice versa.

Mass and energy are different aspects of the same thing

The conversion factor between the unit of mass (the kg) and the unit of energy (the joule, J) is c^2. So, 1 kg of matter has an energy content of

$m_0c^2 = (1kg)*(3x10^8 m/s)^2 = 9x10^{16} J$

This is enough to send a payload of a million tons to the moon.

3. **An electron (m_0 = 0.511 MeV/c^2) and a photon (m_0 =0) both have momenta of 2.00 MeV/c. Find the total energy of each.**
   Solution:
   The electron's total energy is
   $E = \sqrt{m_0^2c^4 + p^2c^2} = \sqrt{(0.511 MeV/c^2)^2c^4 + (2.00 MeV/c)^2c^2} = 2.064\ MeV$
4. **What is the percentage increase in the mass of an electron accelerated to a K.E of 500 MeV? Use rest mass of electron = 0.511 MeV/c^2.**
5. **What is the speed of a particle if its kinetic energy is 1% larger than $\frac{1}{2}m_ou^2$?**

Concept of Ether:

When we say that the speed of sound in dry air at 0°C is 331.3 m/sec, we have in mind an observer, and a corresponding reference system, fixed in the air mass through which the sound wave is moving. It is however known that sound waves are mechanical vibrations which are longitudinal in nature cannot propagate in vacuum and require a medium having some non-zero density.

However, when we say that the speed of light is $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ = 2.997925×10^8 m/s, it is not clear at all what reference system is implied. A reference system fixed in the medium of propagation of light presents difficulties because no medium seems to exist in contrast to sound. According to Maxwell, light waves are electromagnetic wave and the physicists' upto the 19th century felt quite sure that the EM waves require a propagation medium like all other kinds of waves. For light, they assumed this medium named as 'Luminiferous Ether'. But problems arose when Ether is attributed to some strange properties:

i) Since light passes through vacuum, so Ether must be massless and of zero density and must have perfect transparency to account for its undetectability.
ii) Wave propagation requires shearing forces and these forces can occur in solid only. It means that Ether must be a rigid solid filling the whole space. As the velocity of wave propagation depends on the elasticity of the medium, Ether must be highly elastic. Thus, the whole free space must be filled up with such an elastic medium to sustain the vibration of light waves.

Though these are difficult to predict the people believed the Ether medium. A solution of Maxwell's equation gives the speed of light, c = 3×10^8 m/sec, this is in agreement with experimental result. This is now well known that light waves are transverse electromagnetic waves in origin and do not need any medium.

Michelson and Morley mounted the interferometer on a massive stone slab for stability and floated the apparatus in mercury so that it could be rotated smoothly about a central pin. In order to make the light path as long as possible, mirrors were arranged on the slab to reflect the beams back and forth through eight round trips.

The arrangement was capable of measuring 1/100th of a fringe shift. But, actually no fringe shift was found during this measurement. The experiment was performed at different seasons and different places with the same result every time and no fringe shift was detected. Thus, it was concluded that the relative velocity between the earth and the Ether is zero and there is no fringe shift at all.

The result of Michelson-Morley experiment was explained by Einstein and he concluded that:

i) There is no ether frame in space, and
ii) The speed of light in free space is invariant i.e. it is constant and is independent of the motion of source, medium or the observer. This fact lead to n = 0. Downstream and cross-stream speed of light is c, not $|c+v|$.

Conclusion:
I hope I have succeeded in giving you a flavor of Einstein's special theory of relativity. Key points I would like you to take home with you are:

1. The speed of light is constant regardless of the inertial frame in which it is measured. All of the theory's conclusions are derived from this simple **experimental** fact.
2. Simultaneity is a relative concept. The chronological order in which two events occurred may depend on the frame of the observer.
3. Faster than light speed travel or communication is impossible.
   (Otherwise, causality will be broken.)
4. Interesting phenomena like time dilation and Lorentz contraction have been predicted and observed.
5. Equivalence of mass and energy

Famous People:
The following is a list of famous people whose names I have mentioned in my lecture notes.

- Galileo Galilei (1564-1642)
- Sir Isaac Newton (1643-1727)
- James Clerk Maxwell (1831-1879)
- Hendrik Antoon Lorentz (1854-1928)
- Albert Einstein (1879-1955)

1. Why the universe is believed to be expanding?
2. The expansion of universe began about 13 billion years ago. How do you believe the expansion of entire universe?
3. What do you mean by Doppler Effect in light? Find the expression of observed frequency of light in the case of longitudinal Doppler Effect.
4. State Hubble’s law?
5. The relativistic equation for the kinetic energy is $K = mc^2 - m_0c^2$, where the terms have their usual meaning. Find the kinetic energy of the body when it is moving with a very low speed i.e., $v/c << 1$.