Phy 115 (Sound)
Offered: ARCH

Lecture No. 1-3 (review)

Topics: Simple Harmonic Motion and Related derivations, examples

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Review of some earlier topics of the Course

Lectures: 1-5

- Differential equation of simple harmonic oscillation
- Energy of simple harmonic oscillator
- Damped oscillation
- Forced oscillation.

Differential equation of a simple harmonic oscillator

Hook's law, $F=-ky$

F=restoring force, k=force constant

Newton's 2nd law of motion,

$F = mass \times acceleration = m\frac{d^2y}{dt^2}$

Combination of Hook's law and Newton's 2nd law of motion:

$F = -ky = m\frac{d^2y}{dt^2}$

$\Rightarrow m\frac{d^2y}{dt^2} + ky = 0$

$\Rightarrow \frac{d^2y}{dt^2} + \frac{k}{m}y = 0$ (1.1)

$\omega = \sqrt{\frac{k}{m}}$ = angular frequency

$\therefore \frac{d^2y}{dt^2} + \omega^2 y = 0$ (1.2)

Equation (1.2) is the differential equation of a simple harmonic oscillator.

Solution:

$\frac{d^2y}{dt^2} = -\omega^2 y$

$\Rightarrow \int \left(2\frac{dy}{dt}\right)\frac{d^2y}{dt^2}\,dt = -\omega^2 \int \left(2y\frac{dy}{dt}\right)dt$

$\Rightarrow \left(\frac{dy}{dt}\right)^2 = -\omega^2 y^2 + C$ (1.3)

Applying boundary conditions, at $y_{max}=a$, $\frac{dy}{dt} = 0$ $[K.E. = 0]$. So, from equation (1.3) we get, C= $\omega^2 a^2$

Now, $\left(\frac{dy}{dt}\right)^2 = \omega^2 (a^2 - y^2)$

$\Rightarrow \frac{dy}{dt} = \pm\omega\sqrt{(a^2 - y^2)}$

$\Rightarrow \int \frac{dy}{\sqrt{(a^2 - y^2)}} = \omega \int dt$

$\Rightarrow \sin^{-1}\frac{y}{a} = \omega t + \varphi$

$\therefore y = a\sin(\omega t + \varphi)$ (1.4)

Various Equations Related to SHM

Equations of Displacement

- $y = a\sin(\omega t + \varphi)$ (1.4)
- $y = a sin\,\omega t\,cos\,\varphi + a\,cos\omega t\,sin\,\varphi$ (1.5)
- $y = A\,sin\,\omega t + B\,cos\,\omega t$ (1.6)

Assuming,
$acos\varphi = A$
$asin\varphi = B$

- Velocity, $\frac{dy}{dt}=\omega\sqrt{(a^2-y^2)}$ (1.7)
- Maximum velocity, $\left(\frac{dy}{dt}\right)_{\max}=\omega a$ (1.8)
- Acceleration, $\frac{d^2y}{dt^2} = -\omega^2 y$ (1.9)
- Maximum Acceleration, $\left(\frac{d^2y}{dt^2}\right)_{\max}=-\omega^2 a$ (1.10)

- Angular frequency, $\omega=\sqrt{\frac{k}{m}}$ (1.12)
- Time period, $T=2\pi\sqrt{\frac{m}{k}}$ (1.13)
- <u>Initial phase, φ</u>
- If at t=0, y=0, then φ=0; (counting is started at the equilibrium) $y = a\sin\omega t$
- If t=0, y=a, then φ=π/2; (counting is started at the amplitude) y= $a\sin(\omega t + \pi/2) = a\cos\omega t$
- If t=t′, y=*0*, (counting is started before the equ. position) then, y= $a\sin(\omega t - e)$
- If t=t′, y=*0*, (counting is started after the equ. position) then, y= $a\sin(\omega t + e)$

Total energy of a particle executing SHM

- If no non-conservative forces (like friction) act on the oscillator the total energy of the particle will be,

  E= K+U=constant

- Kinetic energy, K=$\frac{1}{2}m\left(\frac{dy}{dt}\right)^2=\frac{1}{2}m[\omega a cos(\omega t+\varphi)]^2$

  $=\frac{1}{2}m\,\omega^2a^2cos^2(\omega t+\varphi)$

  $=\frac{1}{2}ka^2cos^2(\omega t+\varphi)$ $\quad [\because m\omega^2=k]$

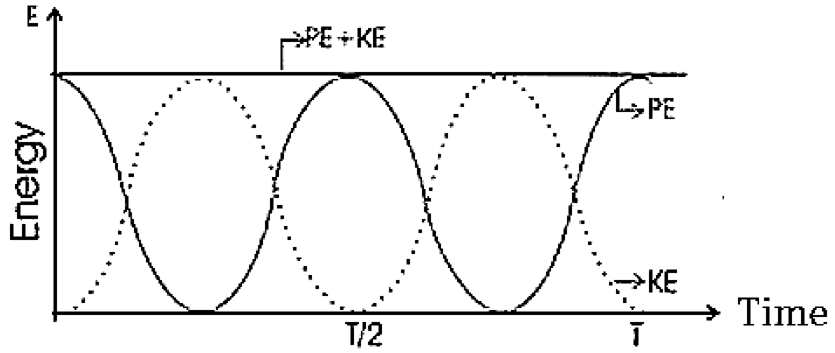
- Potential energy, U=$\int_0^y m\left(\frac{d^2y}{dt^2}\right)dy$

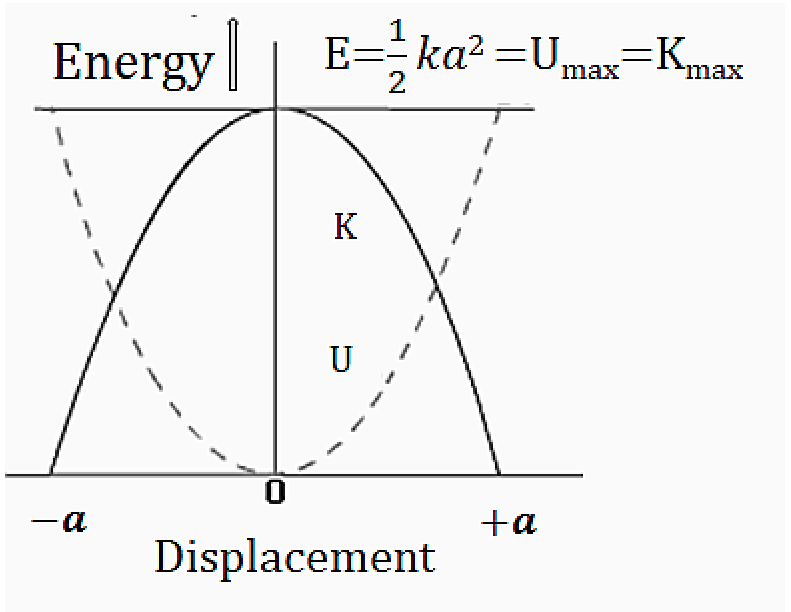
  $=\int_0^y m(\omega^2y)\,dy=\frac{1}{2}m\omega^2y^2$

  $=\frac{1}{2}m\,\omega^2a^2sin^2(\omega t+\varphi)$

  $=\frac{1}{2}ka^2sin^2(\omega t+\varphi)$

- E=$\frac{1}{2}ka^2\,[cos^2(\omega t+\varphi)+sin^2(\omega t+\varphi)]=\frac{1}{2}ka^2=$ constant





Average energy of a particle executing SHM

- Average kinetic energy, K_{avg} $=\frac{1}{T}\int_0^T \frac{1}{2} ka^2 cos^2(\omega t+\varphi)\ dt = \frac{ka^2}{4T}\int_0^T 2cos^2(\omega t+\varphi)\ dt$

$$=\frac{ka^2}{4T}\int_0^T [1+cos2(\omega t+\varphi)]\ dt$$

$$=\frac{ka^2}{4T}\left[\int_0^T dt+\int_0^T cos2(\omega t+\varphi)dt\right]$$

$$=\frac{ka^2}{4T}\,[t]_0^T+\frac{ka^2}{4T}\times 0$$

$$=\frac{1}{4}ka^2$$

- Average potential energy, U_{avg} $=\frac{1}{T}\int_0^T \frac{1}{2} ka^2 sin^2(\omega t+\varphi)\ dt = \frac{ka^2}{4T}\int_0^T 2sin^2(\omega t+\varphi)\ dt$

$$=\frac{ka^2}{4T}\int_0^T [1-cos2(\omega t+\varphi)]\ dt$$

$$=\frac{ka^2}{4T}\left[\int_0^T dt-\int_0^T cos2(\omega t+\varphi)dt\right]$$

$$=\frac{ka^2}{4T}\,[t]_0^T-\frac{ka^2}{4T}\times 0$$

$$=\frac{1}{4}ka^2$$

Spring-mass System

Hook's law for extended spring, $F=-k\Delta l$

K=spring constant, Δl=extension, l=length of the spring [Fig (a)]

From Fig (b)Weight, $mg=k\Delta l$

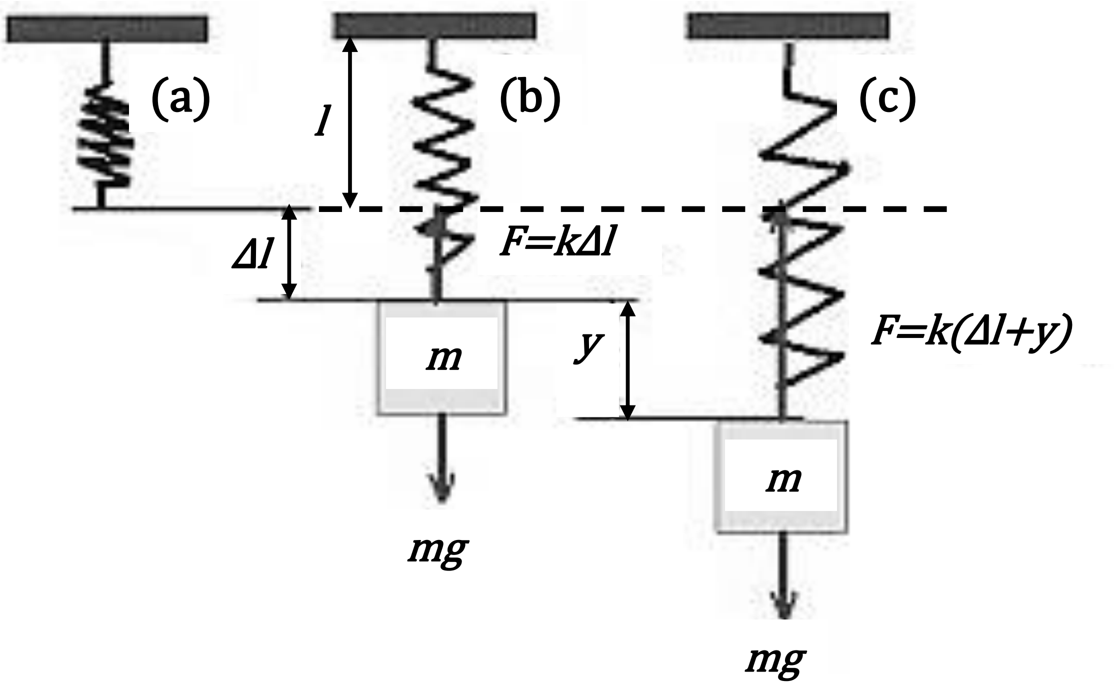
The Fig (c), The upward force the spring exerts on the body is $k(\Delta l+y)$

The downward force acting on the body is mg.

So, The resultant force on the body,

$F=mg-k(\Delta l+y)-=-ky$

Newton's 2^{nd} law of motion gives, $F=m\frac{d^2y}{dt^2}$



Finally, $m\frac{d^2y}{dt^2}=-ky$

$\frac{d^2y}{dt^2}+\frac{k}{m}y=0$; Same as the Diff. equation of SHM.

Time period, $T=2\pi\sqrt{\frac{m}{k}}$

Torsional pendulum

Differential equation:

Hook's law for angular motion,

$$\tau = -\kappa\theta$$

κ=torsional spring constant

Newton's 2nd law for angular motion,

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$$

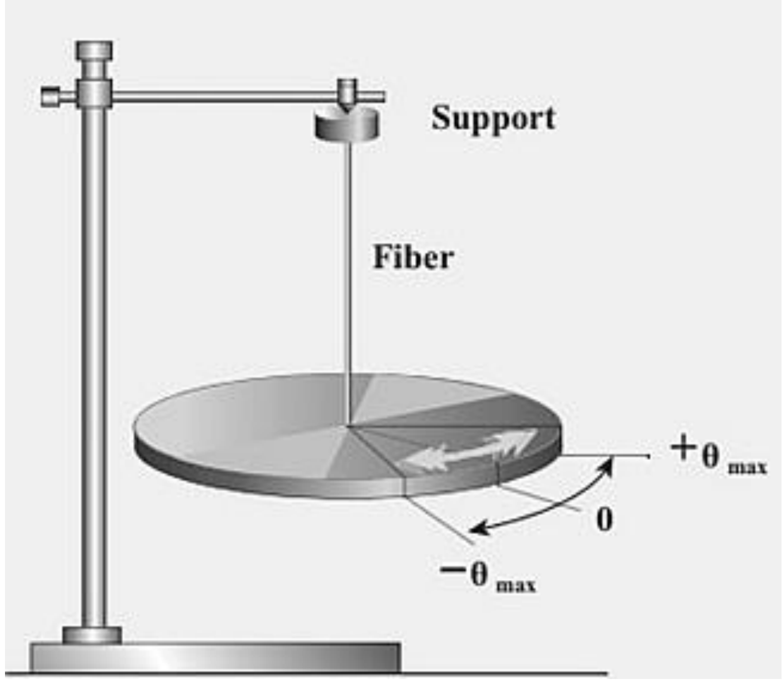
Equating expressions,

$$-\kappa\theta = I\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{\kappa}{I}\theta = 0$$

Solution of the diff. equation

$$\theta = \theta_m sin(\omega t + \varphi)$$



θ = angular displacement

θ_m= angular amplitude

ω = angular frequency = $\sqrt{\frac{\kappa}{I}}$

Time period, T = $2\pi\sqrt{\frac{I}{\kappa}}$

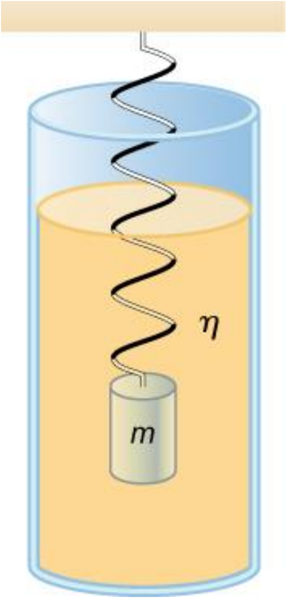
Free Oscillation and Damped Oscillation

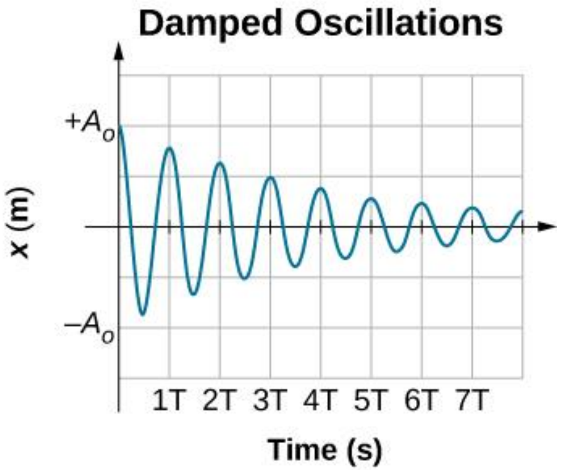
- If a oscillation occurs flawlessly without any resistive force acting on it is called free oscillation.
- Any oscillation occurring in an air medium, experiences frictional force and consequent energy dissipation occurs.
- The amplitude of oscillation decays continuously with time and finally diminishes. Such oscillation is called damped oscillation.
- The dissipated energy appears as heat either within the oscillating system itself or in the surrounding medium.

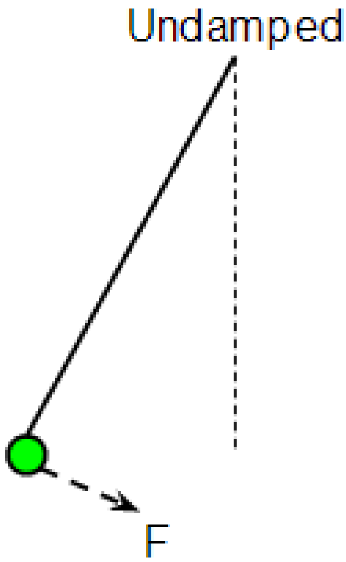
Characteristic of Damped Oscillation

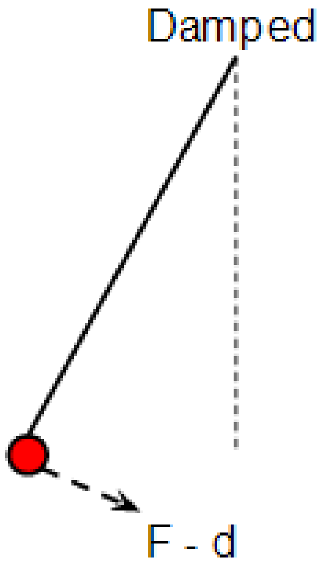
- Frictional force, acting on a body opposite to the direction of its motion , is called damping force.
- Damping force reduces the velocity and the kinetic Energy of the moving body.
- Damping or dissipative forces generally arises due to the viscosity or friction in the medium and are non conservative in nature.
- When velocities of body are not high, damping force is found to be proportional to velocity v of the particle
- The frequency of damped oscillator is always less than that of it's natural or undamped frequency.
- Amplitude of oscillation does not remain constant, rather it decays with time

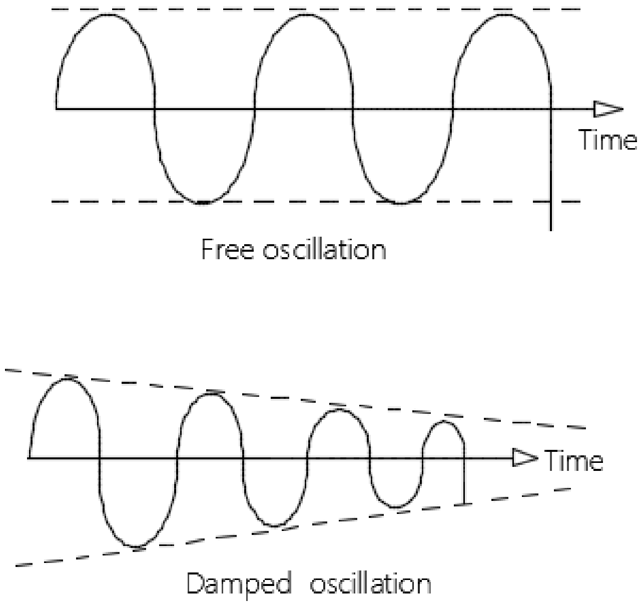
Free Oscillation and Damped Oscillation











Free and damped oscillations

Reference

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