E3: Verification of Biot-Savart law and Tangent law

Theory:

The magnetic field at a point due to a current flowing through a wire is found to vary

i) directly with the strength of the current

ii) inversely as the perpendicular distance of the point from the wire

The relationship between the magnetic field $\vec{B}$ and the current i is given by

$$\oint \vec{B}.\vec{dl} = \mu_o i \tag{1}$$

where μ_o is the magnetic permeability in vacuum and is equal to $4\pi \times 10^{-7}$ $WbA^{-1}m^{-1}$ and *dl* is the smallest element of the wire. This is known as Ampere's law. This law can be used to calculate magnetic field only for the symmetry distribution of the current. However, to compute $\vec{B}$ at any point due to arbitrary current distribution, the relation between $\vec{B}$ and i becomes

$$dB = \frac{\mu_o i}{4\pi} \frac{dl \sin\theta}{r^2} \tag{2}$$

where $d\vec{B}$ is the field due to the current element $id\vec{l}$. This is called Biot-Savart law, where r is the displacement vector from the element $\vec{dl}$ to a point and θ is the angle between $\vec{r}$ and $\vec{dl}$.

Applying Biot-Savart law we can calculate $\vec{B}$ at a distance x from the center of the circular loop of radius R carrying a current i as shown in Fig. 1.

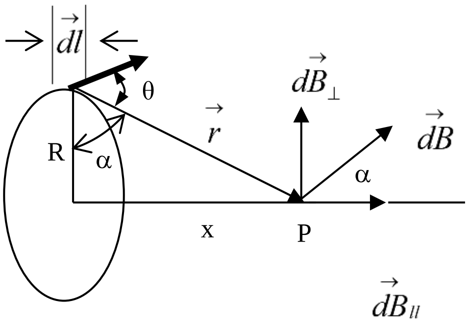

Fig. 1: A circular loop of current. The current element *idl* of the loop sets up a field *dB* at a point P on the.

The field $d\vec{B}$ can be resolved into two components: (i) $\vec{dB}_\perp$ along the axis of the loop and (ii) $\vec{dB}_{11}$ at right angle to the axis. Only the $\vec{dB}_{11}$ contributes to the total induction $\vec{B}$ at the observation point P (Fig. 1). This is because the component $\vec{dB}_{11}$ due to all the current elements $id\vec{l}$ point in different directions perpendicular to the axis, and their resultant for the complete loop is zero.

Thus

$$B = \int \vec{dB}_{II} \tag{3}$$

From Fig. 1 we have $\vec{dB}_{II} = dB\cos\alpha$

Putting the value of *dB* from equation (2), we have

$$\vec{dB}_{II} = \frac{\mu_o i}{4\pi}\frac{dl\sin\theta}{r^2}\cos\alpha \tag{4}$$

Since $\theta = \pi/2$

$$\vec{dB}_{II} = \frac{\mu_o i}{4\pi}\frac{\cos\alpha}{r^2}dl \tag{5}$$

From Fig. 1, we have

$$r = \sqrt{R^2 + x^2} \quad and \quad \cos\alpha = \frac{R}{r}\frac{R}{\sqrt{R^2 + x^2}}$$

Putting these values in equation (5), we obtain

$$\vec{dB}_{11} = \frac{\mu_o i}{4\pi}\frac{R}{\left(R^2 + x^2\right)^{3/2}}dl \tag{6}$$

From equation (3)

$$B = \int \vec{dB}_{II} = \frac{\mu_o i}{4\pi}\frac{R}{\left(R^2 + x^2\right)^{3/2}}\int dl \qquad \because \int dl = 2\pi R$$

or

$$B = \frac{\mu_o i}{2}\frac{R^2}{\left(R^2 + x^2\right)^{3/2}} \tag{7}$$

If N is the number of loops in the coil then equation (7) becomes

$$B = \frac{\mu_o iN}{2}\frac{R^2}{\left(R^2 + x^2\right)^{3/2}} \tag{8}$$

at x = 0,

$$B = \left(\frac{\mu_o N}{2R}\right)i \tag{9}$$

If $K = \left(\frac{\mu_o N}{2R}\right)$, then

$$B = K\,i \tag{10}$$

If a magnetic compass needle is placed freely, it will align itself with the horizontal component of the earth’s magnetic field, $\vec{B}_e$. If there are other magnetic fields around the compass, its alignment will represent the result of multiple fields. When a compass needle is placed in the magnetic field $\vec{B}$, the needle will be deflected from the magnetic meridian to an angle δ as shown in Fig. 2.

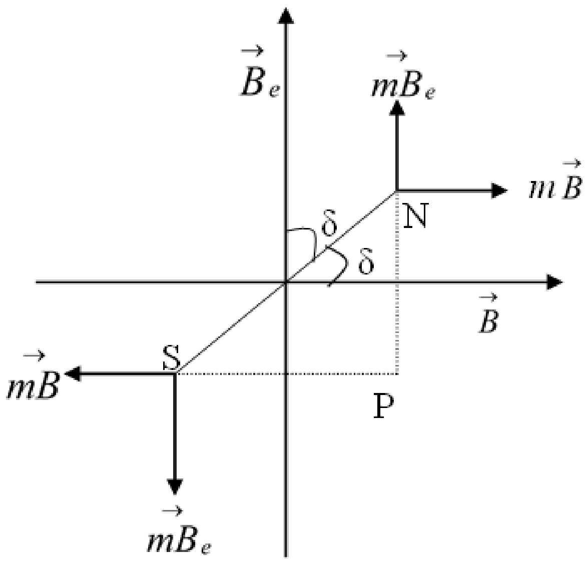

Fig. 2: Deflection of compass needle from the magnetic meridian due to the magnetic field B.

At equilibrium, the deflecting and controlling couples are equal. Thus

$$m\vec{B} \times NP = m\vec{B}_e \times SP$$

where, m is pole strength and $\vec{B}_e$ is the horizontal component of Earth magnetic field.

$$\frac{\vec{B}}{\vec{B}_e} = \frac{SP}{NP} = \tan\delta$$

$$\vec{B} \propto \tan\delta \qquad [\ \vec{B}_{\|} \text{ is constant at a particular place}] \tag{11}$$

$$B = M \tan \delta \tag{12}$$

where, M is the proportional constant.

Using equation (10), the value of B can be evaluated for any particular current i, and the corresponding deflection δ can be obtained from the compass. Thus, the tangent law can be verified from the plot (i.e. straight line) of B versus $tan\delta$.

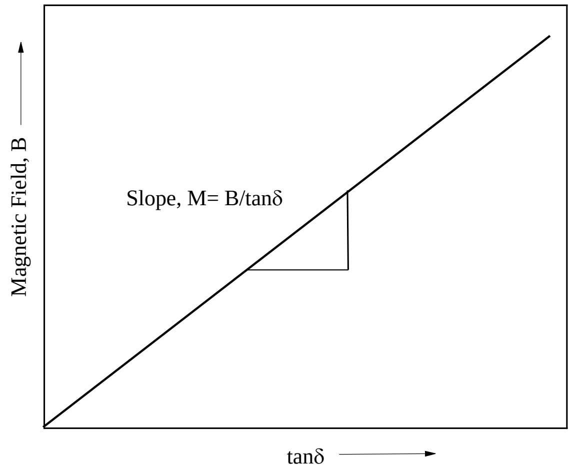

Fig. 3: Magnetic field vs *tanδ graph.*

On the other hand, according to equation (8) $B \propto \frac{1}{x}$. To prove $B \propto \frac{1}{x}$, the magnetic needle will have to be placed at different distances x from the centre of the coil. For a particular position (x) of the needle and for a particular current (i), the value of δ can be observed from the deflection of the needle. But to obtain the value of B at different x, equation (12) can be used. For this, the only value of M is required, as δ is known. From the plot of B vs $\tan\delta$ graph, the slope M can be obtained. Hence a different x values, the different B values can be obtained from equation (12). The plot of B vs x will verify $B \propto \frac{1}{x}$ and hence the Biot-Savart law. The increase of *B* with the increase of current element can be demonstrated by the increase of coil turns N.

Apparatus:

1. Multimeter
2. D.C. power supply
3. A current carrying coil attached in a table
4. Compass needle
5. Resistor

Procedure:

1. Arrange your apparatus as shown in Fig. 1.
2. Before connecting the power supply, place the compass on the table at the center of the coil.
3. Rotate the compass until 0° and 180° align with 50 cm of the meter scale.
4. Rotate table until the compass needle points to North (0°) on the scale (see Fig. 2).

5. Before connecting the power supply, be sure the output regulator knobs in the power supply are at extreme left. **Do not change current knob during the experiment time**.
6. Set the multimeter at 10 amp range.
7. Switch on the power supply and adjust the current *i*, by using voltage knob (keep the current knob at extreme left), at 0.02 amp (approx) and record the deflection δ of the compass needle.
8. Measure δ for at least ten different currents in the circuit (multimeter) within the range from 0 to 2 amp and voltage range from 0 to 15 volt. **Do not exceed current above 2 amp and voltage above 15 volts.**
9. Using voltage knob, adjust the current *i* in the circuit (multimeter) for which the deflection δ of the compass needle is to be about 70°. Record δ at the center of the coil.
10. Keeping current *i* and voltage V constant, move the compass from the center of the coil along the line OB without rotating it, and measure δ for different distances *x* (say at 10 different positions using 3 cm interval).
11. Repeat procedure 9 along the line OA.
12. Draw a graph, *B* along vertical axis and *tanδ* along horizontal axis where value of B is calculated using equation (9) and *tanδ from table 1*. Find the slope M from this graph (see Fig.3).
13. Draw a graph, *B* along vertical axis and *x* along horizontal axis for the data taken in steps 9 and 10 (see Fig.4). Calculate B using equation (12) where value of M is measured in step 12.

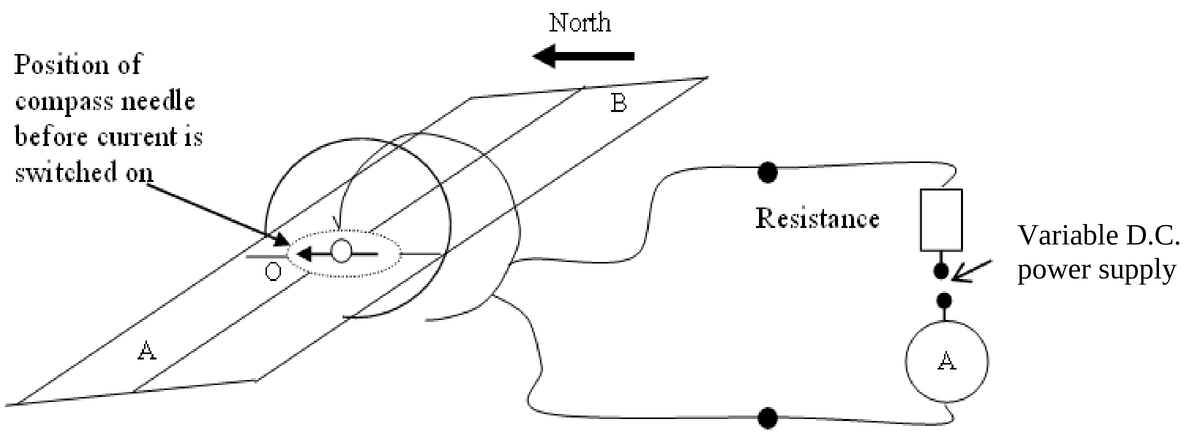

Fig. 3: Circuit diagram to verify Biot-Savart law.

Data Collection:

Radius of the coil, R = (in meter)
Number of turns, N =

Table-1 Data for B and tanδ at center of the coil

No. Observations	Current i (A)	Deflection, δ (degree)	tanδ	Magnetic field, B (Wbm^{-2})
1				
2				
3				
..				
..				
9				
10				

Table-2 Data for B at various distances

No. Observations	x (m)	δ (degree)	tanδ	B=M tanδ (Wbm^{-2})
Along OA				
1				
2				
3				
... ...				
... ...				
9				
10				
Along OB				
1				
2				
3				
... ...				
... ...				
9				
10				

Results:

1. Plot of *B* versus *tanδ* verifies the Tangent law.
2. Plot of *B* versus *x* verifies the Biot-Savart law.

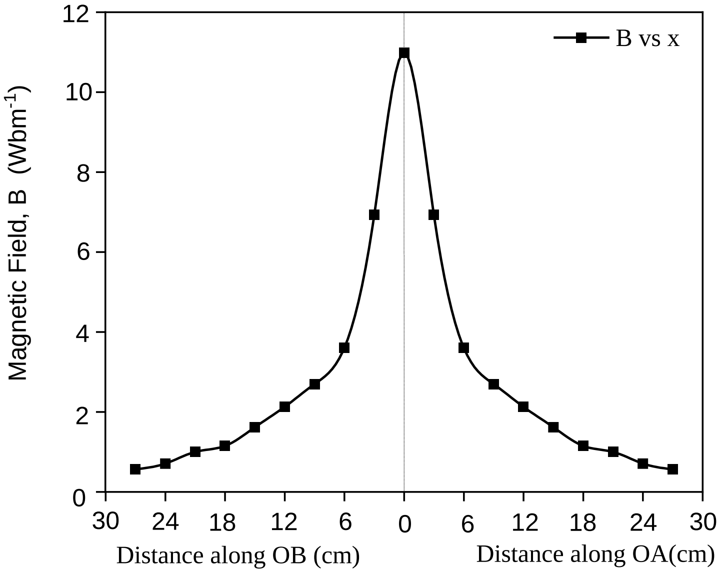

Fig. 4: Magnetic field B vs distance x graph

Discussion: