G_5: Determination of the moment of inertia of a point mass and verification of conservation of angular momentum

I. Determination of the moment of inertia of a point mass

Theory:

Theoretically, the moment of inertia, I, of a point mass is given by $I = MR^2$, where M is the mass, and R is the distance of the mass from the axis of rotation. Since M_1 and M_2 are two masses equidistant from the center of rotation, the total moment of inertia will be

$$I_{total} = M_{total}R^2 \qquad (1)$$

Where $M_{total} = M_1 + M_2$, the total mass of both point masses.

To find the moment of inertia experimentally, a known torque is applied to the object and the resulting angular acceleration is measured. Since $\tau = I\alpha$,

$$I = \tau/\alpha \qquad (2)$$

Where α is the angular acceleration, which is equal to a/r (a = linear acceleration and r = the radius of the chosen pulley about which the thread is wound), and τ is the torque caused by the weight hanging from the thread that is wrapped around the 3-step Pulley. Now,

$$\tau = rT \qquad (3)$$

Where T is the tension in the thread when the apparatus is rotating.

Applying Newton's Second Law of motion for the hanging mass (m_o), we get

$$\Sigma F = m_o g - T = m_o a \qquad (4)$$

For Figure 1.1, Solving for the tension in the thread gives:

$$T = m_o (g - a) \qquad (5)$$

Measuring the angular acceleration of the mass (m), the torque and the linear acceleration can be obtained. Thus, using equations (2), (3) and (5), the moment of inertia can be expressed by the following equation.

$$I = m_o (g/a - 1)\, r^2 \qquad (6)$$

'I' can be determined using equation (6).

Apparatus:

1. PASPORT-Compatible Interface
2. 850 Universal Interface, PASCO Capstone Software
3. Mini-Rotational Accessory
4. Base and Support rod and Long Steel Rod
5. Rotary Motion Sensor
6. Hanging Mass, Calipers
7. Triple Beam Balance

Experimental Setup:

1. Attach a mass on each end of the black colored rod equidistant from the rod center.
2. Tie one end of a thread to a Mass Hanger and tie the other end to one of the levels of the 3-step Pulley on the Rotary Motion Sensor.
3. Mount the rod, on which two masses are attached, on the 3-step pulley placed on the Rotary Motion Sensor.
4. Mount the Rotary Motion Sensor on a support rod and connect it to a PASPORT-compatible interface (550 UNIVERSAL INTERFACE). Make sure that the support rod does not interfere with the rotation of the rod and masses (Figure 1.1).

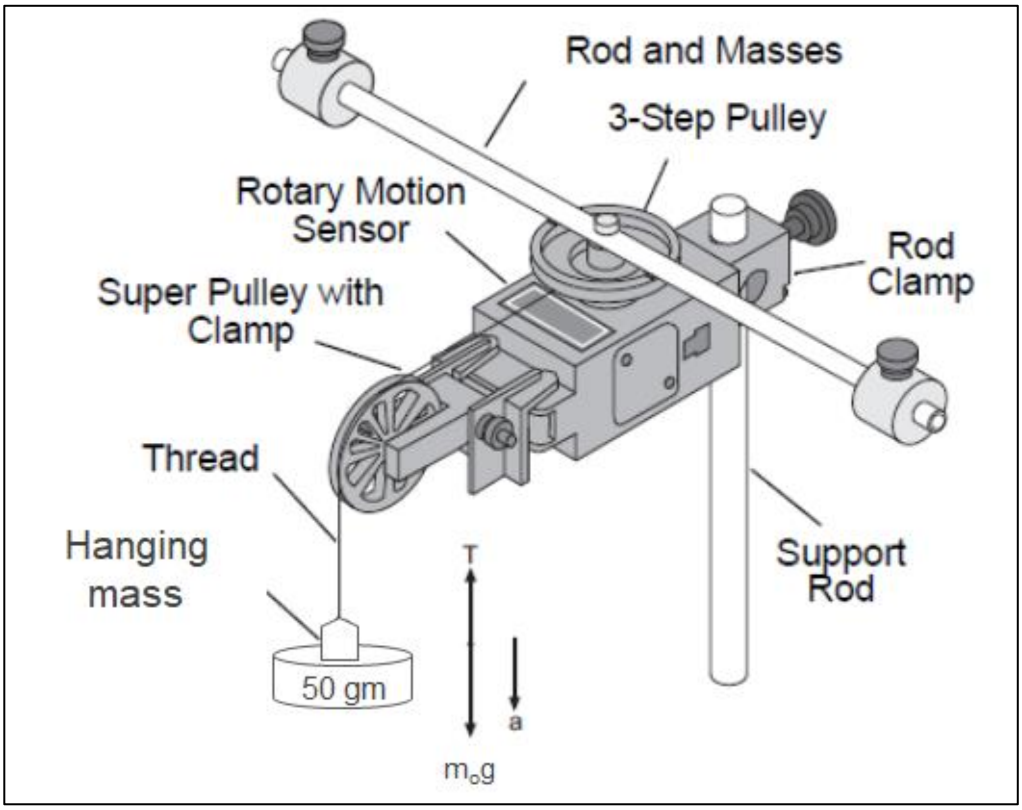

Figure 1.1: Rotary motion sensor and free body diagram

5. Mount the Super Pulley with Clamp on the end of the Rotary Motion Sensor.
6. Drape the thread over the Super Pulley such that the thread is in the groove of the pulley and the hanging Mass hangs freely (Figure 1.1).
7. Keep the hanging mass just under the super pulley.
8. Turn the 3-step Pulley to wind up the thread (wind-up along anti-clockwise direction at the 2^{nd} step of the pulley) so that the hanger is just below the Super Pulley and hold the 3-step Pulley.
9. Connect the Rotary Motion Sensor to the interface and turn the interface on. (Make sure that the interface is connected with the PC).
10. Turn the interface ON.

Procedure:

A. Finding the Acceleration of the Point Masses and Apparatus

1. Open the PASCO Capstone software.
2. In the data acquisition software (PASCO Capstone), select the graph with workbook option. Select "Angular Velocity (rad/s)" (Make sure that you are using the appropriate channel of the interface and the software) for the vertical axis, and "Time (s)" for the horizontal axis both in the graph and workbook.
3. Click Record to begin recording data, and release the 3-step Pulley, allowing the hanger to fall.
4. **Caution! Click Stop to end data recording BEFORE the hanger reaches the floor or the thread completely unwinds from the 3-step Pulley.**
5. In the Graph display, select a region from the linear portion of the angular frequency (velocity) versus time graph (using the graphics menu: Linear mt+b). Export the data table in "CSV" form and save it in the desktop. Record the highlighted data points [selected region] in the Table 1.

Table 1: Data for the plot of angular velocity versus time

Time (s)	Angular velocity (rad/s)

1. In the display, select "Linear" from the curve fit menu. [The slope, m, of the linear fit represents the angular acceleration (α) for the Point Mass and Apparatus] Record the value of the slope, m, as the angular acceleration in Data Table 2.
2. Draw "Angular Velocity (rad/s)" versus "Time (s)" graph in a graph paper, find out the slope, m of the graph and record it in Table 2.

B. Finding the Acceleration of the Apparatus Alone

1. Take the point masses off the ends of the rod.
2. Repeat the procedure from Part A for finding the angular acceleration of the apparatus alone.
3. Record the data in Table 2.

Table 2: Experimental Moment of Inertia Data

Slope, m	From Linear Fit (rad/s^2)	From Graph (rad/s^2)	Average angular acceleration, α (rad/s^2)
Point Mass and Apparatus			
Apparatus Alone			

Calculations

1. Calculate the theoretical moment of inertia of the point masses using the following values in equation (1):
   Total Mass, M_{total} = 150 gm
   The distance of the mass from the axis of rotation, R = 18 cm
2. Calculate the experimental value of the moment of inertia of the point masses and apparatus together using the following values and record the calculation in Table 3.
   Hanging Mass, m_o = 50 gm
   Radius of the chosen pulley, r = 1.65 cm
   Linear acceleration, a = αr
3. Calculate the experimental value of the moment of inertia of the apparatus alone and record the calculation in Table 3.
4. Subtract the moment of inertia of the apparatus from the total moment of inertia of the point masses and apparatus together. Record this in Table 3 as the moment of inertia of the point masses alone.

Table 3: Results

Component	Moment of Inertia (gm-cm^2)
Point Masses and Apparatus Combined	
Apparatus Alone	
Point Masses (experimental value)	
Point Masses (theoretical value)	

5. Calculate the percentage difference to compare the experimental value to the theoretical value using the following equation.
   Percentage difference = $\frac{|V_1 - V_2|}{\left(\frac{V_1 + V_2}{2}\right)} \times 100\%$, V_1 and V_2 are the theoretical and experimental values of moment of inertia, respectively.

II. Verification of conservation of angular momentum

Theory

When a ring is dropped onto the rotating disk, there is no net torque on the system since the torque on the ring is equal and opposite to the torque on the disk. Therefore, there is no change in angular momentum which means angular momentum (L) is conserved. Then, it can be written as:

$$L = I_i \omega_i = I_f \omega_f \tag{7}$$

Where I_i is the initial rotational inertia and ω_i is the initial angular speed of the disk and I_f is the final rotational inertia and ω_f is the final angular speed of the disk and the ring together.

The initial rotational inertia is that of the disk is

$$I_i = I_d = \frac{1}{2} M_1 R^2 \tag{8}$$

The final rotational inertia of a disk and ring together is

$$I_f = \frac{1}{2} M_1 R^2 + \frac{1}{2} M_2 (R_1^2 + R_2^2) \tag{9}$$

Where M_1 is the mass of the disk, M_2 is the mass of the ring, R is the radius of the disk, and R_1 and R_2 are the inner and outer radii of the ring.

Based on the equations (7), (8) and (9), we get

$$[M_1 R^2 + M_2 (R_1^2 + R_2^2)] \omega_f = M_1 R^2 \omega_i \tag{10}$$

In this experiment, ω_i and ω_f are to be determined from the graph of angular velocity versus time. Conservation of angular momentum will be verified experimentally if both sides of the equation (10) are equal.

Apparatus:

1. Rotary Motion Sensor
2. Rotational Inertia Accessory
3. Calipers
4. Large Rod Stand
5. Disk and ring

Experimental Setup:

1. Mount the Rotary Motion Sensor to a support rod and connect it to the Interface (Figure 3.1).
2. Place the disk directly on the pulley.

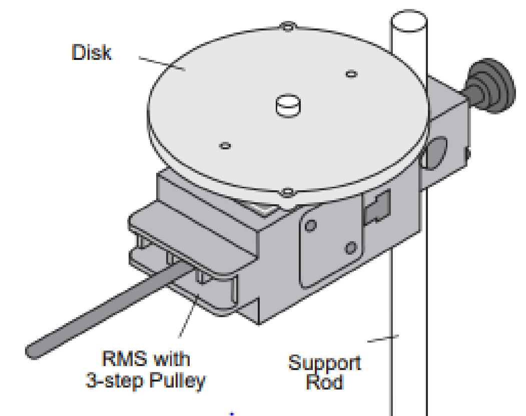

Figure 3.1: Setup for Dropping Ring onto Disk

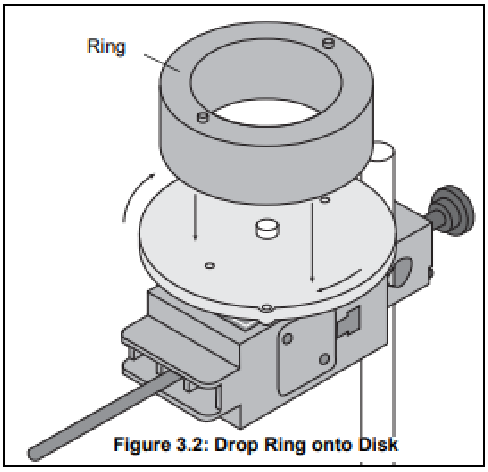

Figure 3.2: Drop Ring onto Disk

Procedure:

1. In the data acquisition software, create an experiment to measure the angular velocity (in rad/sec) versus time (sec).
2. In the data acquisition software (PASCO Capstone), select the graph with workbook option. Select "Angular Velocity (rad/s)" for the vertical axis, and "Time (s)" for the horizontal axis both in the graph and workbook.
3. Hold the ring (Black coloured) with the pins facing up just above the center of the disk.
4. Give the disk a clockwise spin with your hand and click Record to begin recording data.
5. After about 2 seconds, drop the ring on the spinning disk. See (Figure 3.2).
6. Click Stop to end data recording after the disk and ring have made a few rotations.
7. In the Graph display, select the region of the data that represents when the ring was dropped onto the disk. Export the data table in "CSV" form and save it in the desktop. Record the highlighted data [selected region] points in the Table 4.

<u>Table 4: Data for the angular velocity versus time graph</u>

Time (s)	Angular velocity (rad/s)

8. Draw the angular velocity versus time graph in a graph paper using Table 4. Find the coordinates of the point in the plot (beginning of the sharp fall of the graph) that is immediately before the collision. Record the Angular Velocity (ω_i) at this point as the initial angular velocity in Table 5.
9. Find the data point (ending of the sharp fall of the graph) immediately after the collision. Record the Angular Velocity at this point as the final angular velocity (ω_f) in Table 5.

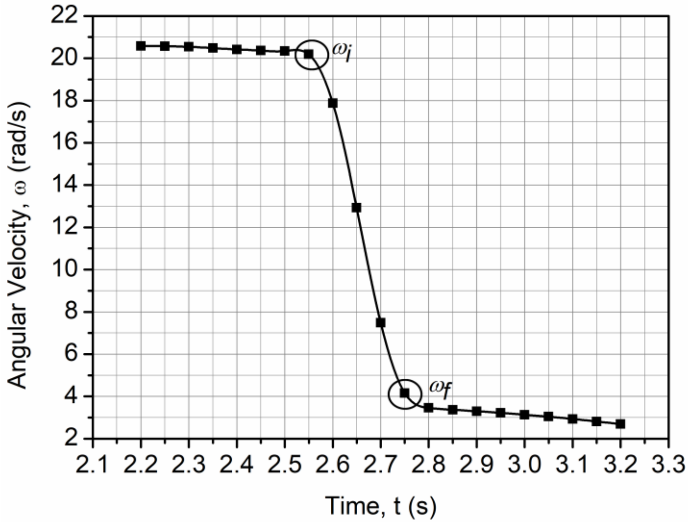

Table 5

Initial angular velocity (from the graph), ω_i (rad/s)	
Final angular velocity (from the graph), ω_f (rad/s)	

10. Calculate initial rotational inertia (I_i) and final rotational inertia (I_f) using the following values in equations (8) and (9).

    Mass of disk, M_1 = 120 gm

    Radius of disk, R = 4.78 cm

    Mass of ring, M_2 = 471 gm

    Inner radius of ring, R_1= 2.75 cm

    Outer radius of ring, R_2 = 3.80 cm

11. Verify conservation of angular momentum putting I_i, ω_i, I_f, ω_f values in equation (10).