W$_1$: Determination of line frequency by Lissajous figures using an oscilloscope and a function generator and verification of the ‘calibration’ of the TIME/DIV Knob at a particular position for different frequencies.

Theory:

A. Consider two simple harmonic motions of the same frequency (ω) but of different amplitudes, a and b, acting on a particle mutually perpendicular to one another (i. e., one vibrates along the X-axis, the other vibrates along the Y-axis). Also, consider the phase difference of these two waves is 90^0. Then,

$$x = a\sin\ wt \quad \text{...} \quad (1)$$

and

$$y = b\ \cos\ wt \quad \text{...} \quad (2)$$

From equations (1) and (2)

$$\frac{x}{a} = \sin\ wt \quad \text{...} \quad (3)$$

$$\frac{y}{b} = \cos\ wt \quad \text{...} \quad (4)$$

From equations (3) and (4)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{...} \quad (5)$$

This is the equation of an ellipse.

If a = b, i.e., the amplitudes of the two vibrations are equal, then equation (5) reduces to

$$x^2 + y^2 = a^2 \quad \text{...} \quad (6)$$

Which is the equation of a circle of radius a.

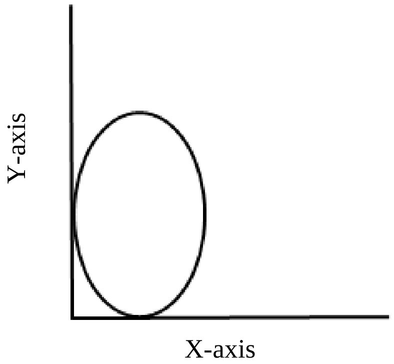

Fig-1: An ellipse

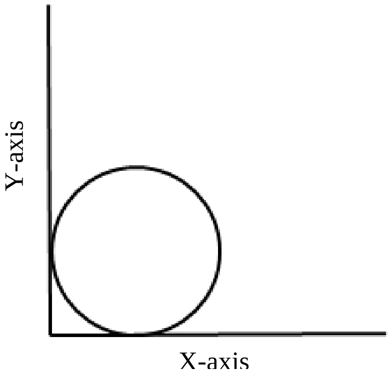

Fig-2: A circle

From equation (5) and (6), we obtain one loop such as an ellipse or a circle, which touches X and Y-axes at one point (Fig-1 and Fig-2).

If we consider frequencies of two waves along X and Y-axes as f_x and f_y, then the frequency ratio is:

$$\frac{f_x}{f_y} = \frac{\omega}{\omega} = 1 \quad \text{...} \quad (7)$$

If the frequencies f_x & f_y are respectively 2ω and ω then the combination of equation (1) and (2) will lead to:

$$\frac{x^2}{a^2} + \frac{4y^2}{b^2}\left(\frac{y^2}{b^2} - 1\right) = 0 \quad \text{.............................} \quad (8)$$

The above equation represents a two-loop pattern (Fig-3), which touches on Y-axis and one-point on X-axis at one point and Y-axis more than one point.

In these cases, the frequency ratio:

$$\frac{f_x}{f_y} = \frac{2\omega}{\omega} = 2 \quad \text{...} \quad (9)$$

If we increase the frequency ratio successively, then we can, in general, write:

$$\frac{f_x}{f_y} = n \quad \text{...} \quad (10)$$

where n= 1, 2, 3, etc. (n= number of loops touches on Y-axis)

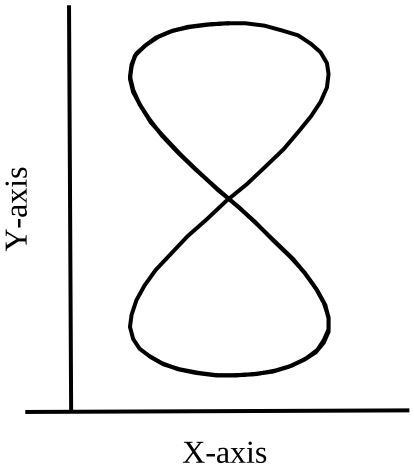

Fig-3

B. Time base T_B of an oscilloscope is the time taken for the cathode ray to sweep unit division of the screen.

$$T_B = \frac{\textit{Time taken to show a complete cycle}}{\textit{No. of division on the screen showing complete cycle}}$$

$$T_B = \frac{\textit{time period}}{d} = \frac{1}{\textit{frequency} \times d} = \frac{1}{fd} \quad \text{.....} \quad (11)$$

If the time/div knob is kept at a particular position then for different frequencies from the function generator, we can measure the number of divisions d between two consecutive peaks, i.e. for a full sine wave displayed on the screen. If we plot frequency versus 1/d, a straight line will be obtained. The value of the slope will give T_B at the particular position of the time/div knob.

Procedure-A

1. Connect the **output** of the signal generator to CH-1 of the oscilloscope (By a cable having two crocodile clips at the end).
2. Connect the **output** of the transformers to CH-2 of the oscilloscope (by a similar cable).
3. Switch **ON** the power of the oscilloscope, signal generator and transformer.
4. Press **sine wave (~) button** of the signal generator. Set the frequency 50 Hz from the signal generator (you will see two sine waves on the display of the oscilloscope).

Additional instructions: If the sine wave does not appear after step 4, follow the procedure mentioned below:

i. Disconnect the **output** of the transformer from oscilloscope.
ii. Press **Main** button of the oscilloscope.
iii. Press **Autoset** button of the oscilloscope.
iv. Adjust the **TIM/DIV** knob and **VOLTS/DIV** knob on the oscilloscope at a suitable position so that you can measure the peak-to-peak distance of the sine wave displayed on the oscilloscope screen.

5. Superimpose two waves by shifting **vertical adjusting knobs** of the oscilloscope (at the middle position)
6. Press **XY-button** of the oscilloscope (don’t touch on the screen. See the “button” just right to the …of the display).
7. Note the reading of the frequency (f_x) from the function generator and n from the oscilloscope and then calculate the line frequency (f_y) from equation (10).
8. Repeat procedure (7) for at least six different frequencies of the function generator so that you will see an increasing of loops (say 1, 2, 3, etc.) on the display with increasing frequency.
9. Draw a graph with frequency (f_x) along the X-axis and the number of loops (n) along the Y-axis. The resulting graph should be a straight line passing through the origin.
10. The inverse of the slope from this graph will give you the average line frequency.

Procedure-B

1. Disconnect the **output** of the transformer from oscilloscope.
2. Press **Main** button of the oscilloscope.
3. Press **Autoset** button of the oscilloscope.
4. Adjust the **TIM/DIV** knob and **VOLTS/DIV** knob on the oscilloscope at a suitable position so that you can measure the peak-to-peak distance of the sine wave displayed on the oscilloscope screen.
5. Set a minimum frequency in the function generator for which you can see a sine wave with two peaks on the display. Now measure the scale divisions between the two consecutive peaks “d”.
6. Repeat the operation-5 for different frequencies (at least 5 different frequencies).
7. Draw a graph with frequency along the X-axis and 1/d along Y-axis. The slope of the graph will give you the time base value.

Remarks: Signal Generator takes time to give a stable reading. Keep waiting until it gives a stable value.

Table-1: Data for the line: frequency

No. of obs.	Frequency in function generator f_x (Hz)	No. of loops touches in Y axis (n)	Line frequency f_y (Hz)	Average line frequency (Hz) (from graph)

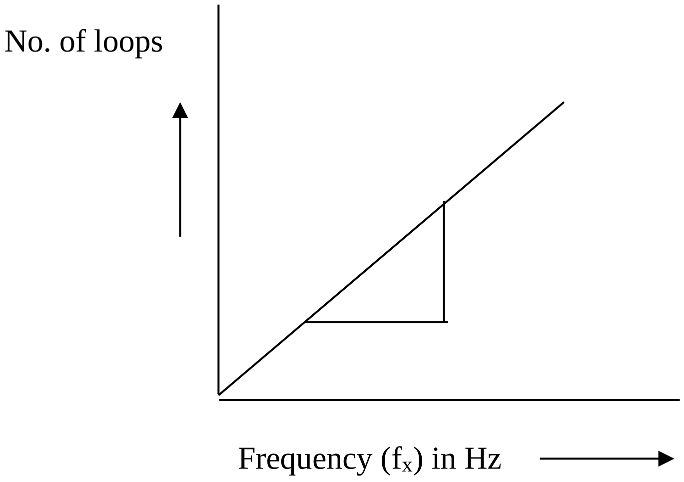

Table-2: Data for verification of the calibration of TIME/DIV knob

No. of obs.	Frequency in function generator f_x (Hz)	Peak to peak distanced (div/volt)	$\frac{1}{d}$	$T_B = \frac{1}{fd}$ in ms (from graph)