M_3: Verification of Heisenberg’s uncertainty principle using single slit diffraction pattern.

Principle

The distribution of intensity in the Fraunhofer diffraction pattern of a slit is measured. The results are evaluated both from the wave pattern view point, by comparison with Kirchhoff's diffraction formula, and from the quantum mechanics standpoint to confirm Heisenberg's uncertainty principle.

Benefits

- Quantum mechanical versus wave theory viewpoints are discussed observing light transversing a slit.
- A measured diffraction pattern can be explained by quite different theories.
- The famous "uncertainty principle" can be confirmed in this setup.

Tasks

1. To measure the intensity distribution of the Fraunhofer diffraction pattern of a single slit (e. g. 0.1 mm). The heights of the maxima and the positions of the maxima and minima are calculated according to Kirchhoff’s diffraction formula and compared with the measured values.
2. To calculate the uncertainty of momentum from the diffraction patterns of single slits of differing widths and to confirm Heisenberg’s uncertainty principle.

Learning outcomes

- Diffraction
- Diffraction uncertainty
- Kirchhoff’s diffraction formula
- Measurement accuracy
- Uncertainty of location
- Uncertainty of momentum
- Wave-particle duality
- De Broglie relationship

Theory

Observations of light passing through narrow openings show that light spreads out behind the opening and forms a distinct pattern on a distant screen. By scanning the pattern with a light sensor and plotting light intensity versus distance, differences and similarities between interference and diffraction are examined. Observation of diffraction intensity can be used in a simple quantum mechanical treatment to confirm the Heisenberg's uncertainty principle.

When diffraction of light occurs as it passes through a slit, the angle to the minima (dark spots) in the diffraction pattern is given by

$$d \sin\alpha = n\lambda \qquad (n = 1,2,3, \ldots) \qquad (1)$$

where$d$is the slit width, α is the angle from the center of the pattern to the n^{th} minimum, λ is the wavelength of the light, and m' is the order of diffraction (1 for the first minimum, 2 for the second minimum, ...counting from the center out).

In Figure 1, the diffraction pattern is shown just below the computer intensity versus position graph. The angle α is measured from the center of the single slit to the first minimum, so n= 1 for the situation shown in the diagram.

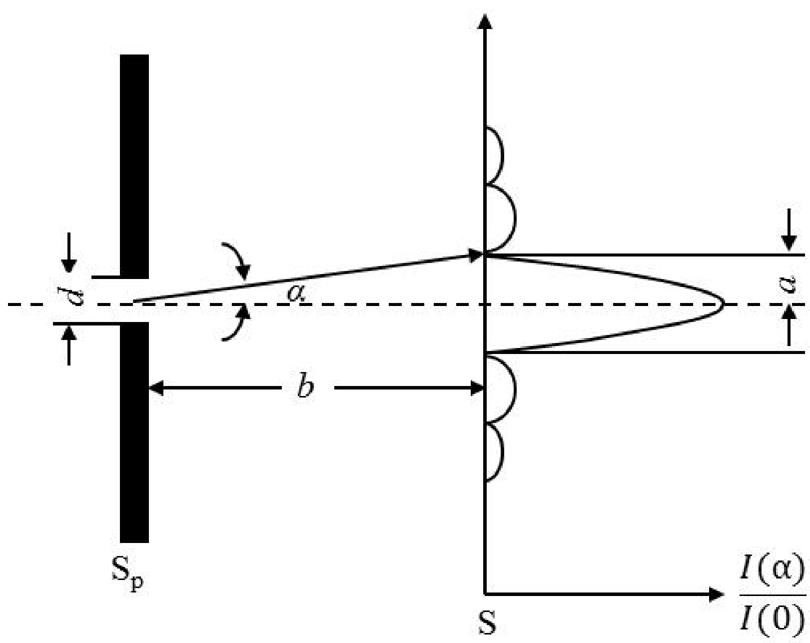

Fig. 1: Fraunhofer diffraction from a single-Slit.(S_p= aperture or slit, S = screen).

Intensity in the diffraction pattern

Light intensity is proportional to the square intensity of the wave producing it (E^2, where$E$ is the amplitudeof the electric field intensity). By using the concepts of superposition of waves [1],we obtain:

$$I(\alpha) = I(0).\left(\frac{\sin\beta}{\beta}\right)^2, \qquad (2)$$

$$\text{Where } \beta = \frac{\pi d}{\lambda}\sin\alpha \qquad (3)$$

The first intensity minima are at $\alpha_n = \sin^{-1}\frac{n\lambda}{d}$ according to Eq. 1.

Quantum mechanical interpretation

The Heisenberg uncertainty principle states that the simultaneous measurements of the momentum and position (or theenergy and time) for a moving particle entails a limitation on the precision (standarddeviation) of each measurement. Namely: the more precise the measurement of position is the more imprecise the measurement of momentum, and vice versa. In themost extreme case, absolute precision of one variable would entail absolute imprecisionregarding the other.If we consider a bunch of photons characterized by position uncertainty Δy andmomentum uncertainty Δp, we can express Heisenberg's relation as:

$$\Delta y \cdot \Delta p \geq \frac{h}{4\pi} \tag{4}$$

Where $h = 6.6262 \times 10^{-34}$ J.s is Planck's constant.

For a bunch of photons passing through a slit of width d we have

$$\Delta y = d \tag{5}$$

In order to estimate Δp, we assume that photons reaching the slit move only in thedirection perpendicular to the slit (x-direction), but after passing through the slit, they willhave velocity components in both directions (x and y). The probability density for the velocity component of photon in the y direction, v_y is given by the intensity distribution in the diffraction pattern.

Defining by α_1 the angle of the first minimum of diffraction (Fig. 1), we can express theuncertainty of velocity and momentum as:

$$\Delta v_y = c \sin \alpha_1, \qquad \text{(where } c \text{ is the speed of light)} \tag{6}$$

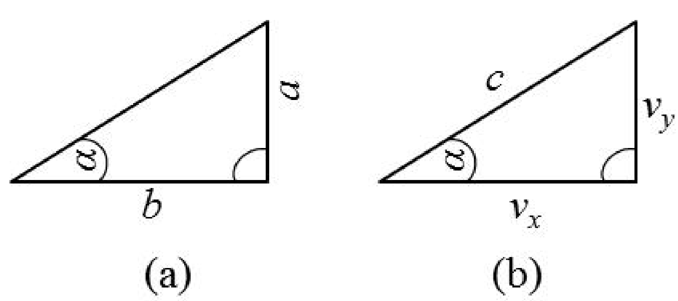

Fig. 2: Geometry of diffraction at a single slit. a) path covered and b) velocity component of a photon.

The uncertainty of momentum is therefore

$$\Delta p_y = m.\ c \sin \alpha_1$$

where$m$ is the mass of the photon and$c$ is the velocity of light. The momentum and wavelength of a particle are linked through the de Broglie relationship:$p= mc= \frac{h}{\lambda}$. Thus

$$\Delta p_y = \frac{h}{\lambda} \sin \alpha_1 \tag{7}$$

According to Eq. 1 the angle α_1 of the first diffraction minimum is therefore

$$\sin\alpha_1 = \lambda/d \tag{8}$$

If we substitute (8) in (7) and use (5) we obtain the uncertainty relationship

$$\Delta y . \Delta p_y = h. \tag{9}$$

If the slit width $\Delta y = d$ is smaller, the first minimum of the diffraction pattern occurs at larger angles α_1 and we can obtain from the position of the first minimum (Fig. 2a) using the relation

$$\tan \alpha_1 = \frac{a}{b}. \tag{10}$$

If we substitute (10) in (7) we obtain

$$\Delta p_y = \frac{h}{\lambda} \sin\left(\tan^{-1}\frac{a}{b}\right) \tag{11}$$

Substituting (5) and (11) in (9) and dividing both side by h we get

$$\frac{d}{\lambda}\sin\left(\tan^{-1}\frac{a}{b}\right) = 1 \tag{12}$$

And therefore the uncertainty principle is verified.The results of the measurements confirm (12) within the limits of error.

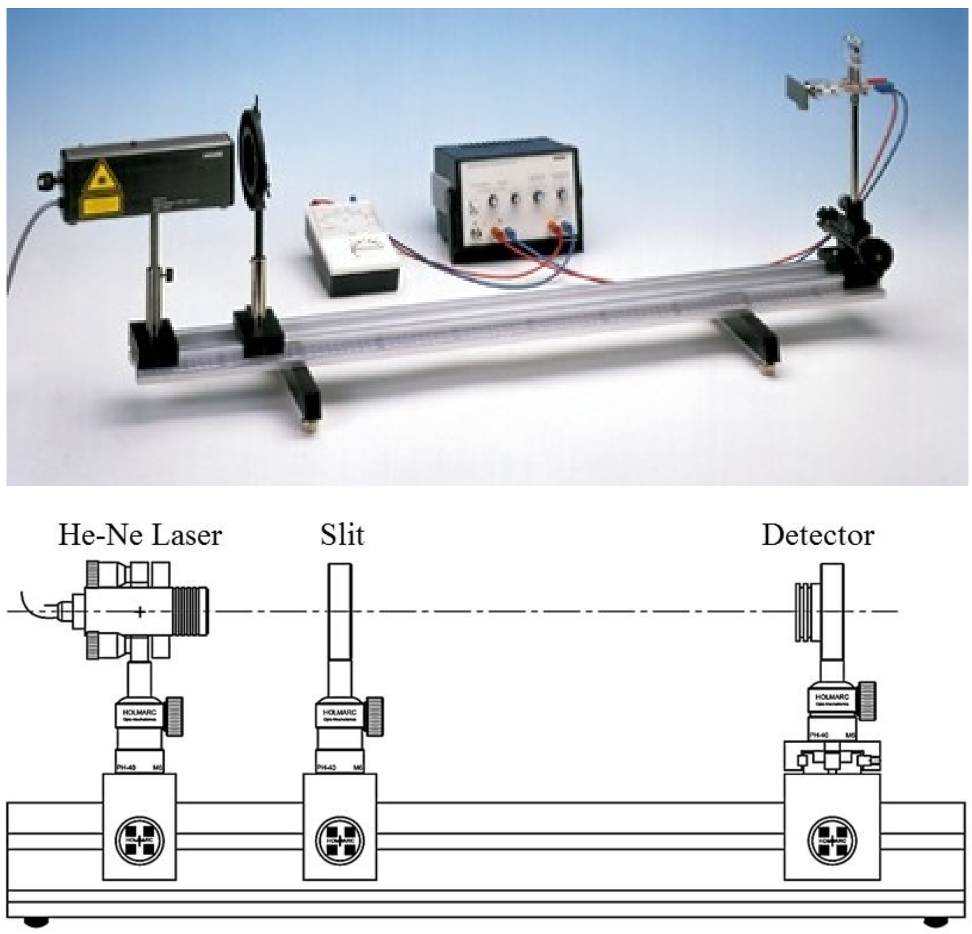

Figure 3: Experimental Setup

Apparatus:

- He-Ne Laser, 0.2/1.0 mW, 230 V AC, Wavelength, $\lambda = 633$ nm
- Optical bench, $L = 1500$ mm
- Base for optical bench, adjustable
- Slide mount for optical bench, $h = 30$ mm
- Diaphragm, 3 single slits
- Sliding device (horizontal)
- Photodetector with Amplifier
- Control unit for Photodetector
- Digital multimeter
- Connecting cords,
- Adapter, BNC-plug/socket
- Diaphragm holder

Procedure:

1. Set up the three slide mounts on the optical bench.
2. Set up the laser beam on the left side of the optical bench on a slide mount.
3. Place the slit on diaphragm front of the laser beam.
4. Set up the photodetector by a slide mount on the optical bench on the right side.
5. Supply power to the laser gun and switch it on by rotating key to position 1 (clockwise).
6. Align the three devices (laser beam, diaphragm, and the photo detector by rotating the screw of the sliding device.
7. Turn off the power supply to the laser gun and connect chord-1 in photo detector of the photoamplifier connect another chord to the multimeter and amplifiers.
8. Multimeter should be kept in the range of 20 or 200 mA and photoamplifier in the DC mode 60 kHz.
9. Power the photo-amplifier by an adaptor.
10. Now, switch on all the devices and take readings of multimeter in different position of the maxima and minima by screwing sliding device. You can start taking reading from 3rd or 4th minimum position of one side to another side of the pattern.
11. Sketch the graph current, *I*(mA) verses distance, *x* (mm) and determine the width of the central Maxima.

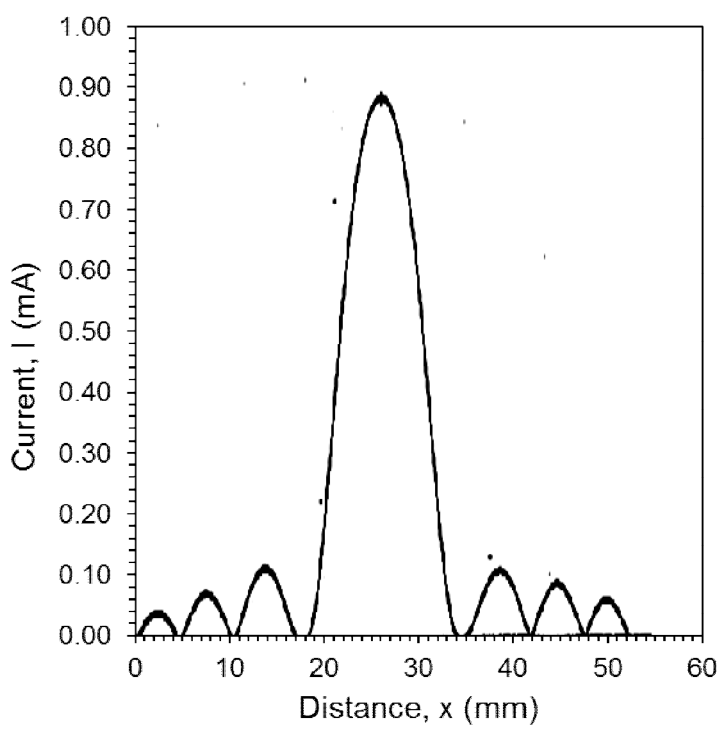

Figure 4: Diffraction pattern from a single slit (Current versus distance curve).

Data Collection:

Distance, *x* (mm)	Current, *I* (mA)	Distance, *x* (mm)	Current, *I* (mA)	Distance, *x* (mm)	Current, *I* (mA)
0.0		0.6		1.2	
0.1		0.7		1.3	
0.2		0.8		1.4	
0.3		0.9		1.5	
0.4		1.0		1.6	
0.5		1.1		etc.	

Calculation:

Calculate from your data using the formula $\frac{d}{\lambda}$ sin ($tan^{-1}\frac{a}{b}$)

Results: Check whether Heisenberg's uncertainty principle is verified or not.

Discussion:

Measure the half width of the central maximum for single slit width (*d*). Measure the distance between the light sensor aperture and the laseraperture (*b*).Verify Heisenberg's uncertainty principle by using this experimental verification(Eq.12). Carefully estimate the errors and discuss about Heisenberg's uncertainty principle.

Caution: Never look directly into a non-attenuated laser beam. After the completionof the all procedure,at first switch off the laser beam rotating the key to'0' position (counter clockwise), power off the laser beam, photo amplifier, turn off the multimeter, disconnect the chord from all the devices like multimeter, photo amplifier, adaptor and photo detector, etc.

Answer the following Questions:

1. What physical quantity is the same for the single slit and the double slit?
2. How does the distance from the central maximum to the first minimum in the single-slit pattern compare to the distance from the central maximum to the first diffraction minimum in the double-slit pattern?
3. What physical quantity determines where the amplitude of the interference peaks goes to zero?
4. In theory, how many interference maxima should be in the central envelope for a double slit with d = 0.25 mm and a = 0.04 mm?
5. How many interference maxima are actually in the central envelope?

References:

[1] A.P. French: Vibrations and Waves, Norton Publ. 1971, Chapter 8, p. 280-297

[2] PASCO Interference and Diffraction EX-9918 guide (written by Ann Hanks)

[3] PHYWE Series of Publications, Laboratory Experiments, Physics LEP 2.3.01, PHYWESYSTEME GMBH