VL-W1: DETERMINATION OF THE SPRING CONSTANT AND THE EFFECTIVE MASS OF A SPIRAL SPRING

Objectives:

- To explore how the time of vertical oscillation depends on the load and how mass of the spring effect on the oscillation.
- To determine the spring constant and the effective mass of a spiral spring.

System requirements:

Computer (Desktop/Laptop), Operating system: Windows, masses-and-springs_en.html file, latest version of web browser (Example: Google Chrome).

Advise:

Students are advised to follow the **procedures written in this manual very strictly** while performing the experiment. D**o not try to explore anything else during the experiment.**

Theory:

If a spring be clamped vertically at the end P as shown in Fig. 1 and is loaded with a mass m_o at the other end A is set into vibration, then the period of vibration of the spring along a vertical line is given by

$$T = 2\pi\sqrt{\frac{m_o+m'}{k}} = 2\pi\sqrt{\frac{M}{k}} \tag{1}$$

Where m' = a constant, called the effective mass of the spring and k = the spring constant, the ratio between the added force and the corresponding extension of the spring.

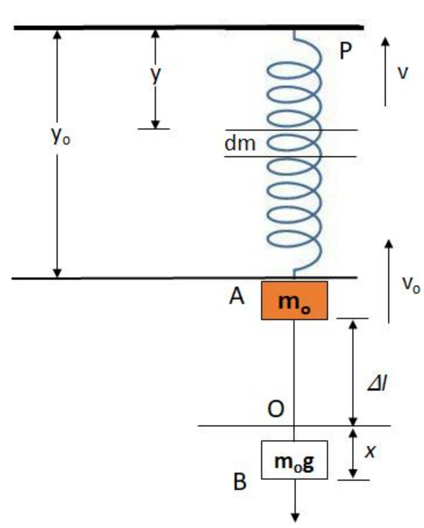

Fig. 1 Determination of the spring constant and effective mass of a given spiral spring.

Consider the kinetic energy of a loaded spring undergoing simple harmonic motion. At the instant under consideration let the load m_o be moving with velocity v_o as shown in Fig. 1. At this same instant a mass element 'dm' of the spring will also be moving up with a velocity v (where $v < v_o$). It is evident that the ratio between v and v_o is just the ratio between the displacements of the masses m_o and dm indicated by y and y_o, respectively.

$$\frac{v}{y} = \frac{v_o}{y_o} \quad \Rightarrow v = \frac{v_o}{y_o}\, y \tag{2}$$

If the mass of the spring is m along with length y_o and it can be written as from Fig. 1.

$$\frac{dm}{m} = \frac{dy}{y_o} \qquad (3)$$

The kinetic energy of the spring will be given by

$$K_s = \int_0^{y_o} \frac{1}{2} v^2 dm \qquad (4)$$

Using equations (2) and (3) and equation (4) becomes,

$$K_s = \frac{1}{2} \int_0^{y_o} \left(\frac{v_o}{y_o}\ y\right)^2\ m \frac{dy}{y_o}$$

$$\Longrightarrow K_s = \frac{1}{2} \frac{v_o^2}{y_o^3} m \frac{y_o^3}{3}$$

$$\Longrightarrow K_s = \frac{1}{2} \left(\frac{m}{3}\right) v_o^2 \qquad (5)$$

The total kinetic energy of the system will then be

$$K_{total} = K_{m_o} + K_s$$

$$\text{Thus} \quad K_{total} = \frac{1}{2} \left(m_o + \frac{m}{3}\right) v_o^2 \qquad (6)$$

The effective mass of the spring, $m' = \frac{m}{3}$ and $m =$ the true mass of the spring

In order to find the spring constant (k), consider the applied force $m_o g$ is proportional to the extension Δl within the elastic limit. Therefore $mg = k\Delta l$, the spring constant will be given by $= \frac{mg}{\Delta l}$. The Δl vs m graph helps finding the slope = $\frac{\Delta l}{m}$. The spring constant could be written as

$$k = \frac{g}{slope} \qquad (7)$$

Procedures:

1. Unzip "**masses-and-springs_en**" file. Now open the "**masses-and-springs_en.html**" file with your web browser say using "Google Chrome". Choose the option "Lab" from the home page of the experiment (Fig 2).

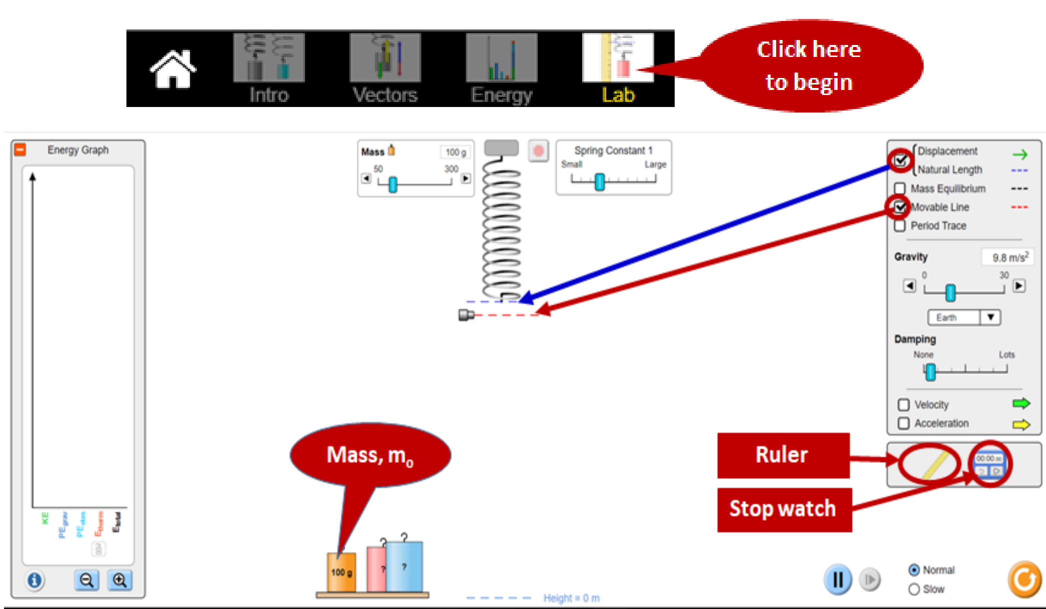

Fig. 2

2. Click on "**movable line**" and "**Displacement/Natural length**" in the list on the right side of the spring (Fig. 2). The "**movable line**" is to be used as a pointer while measuring the extension of the loaded spring.
3. In this experiment only the mass, m_o (**orange color**) indicated in Fig. 2 should be used.

4. Drag the ruler and stop watch from the list on the right side of the spring (Fig. 2) and set the scale on the position as shown in the image (Fig. 3).

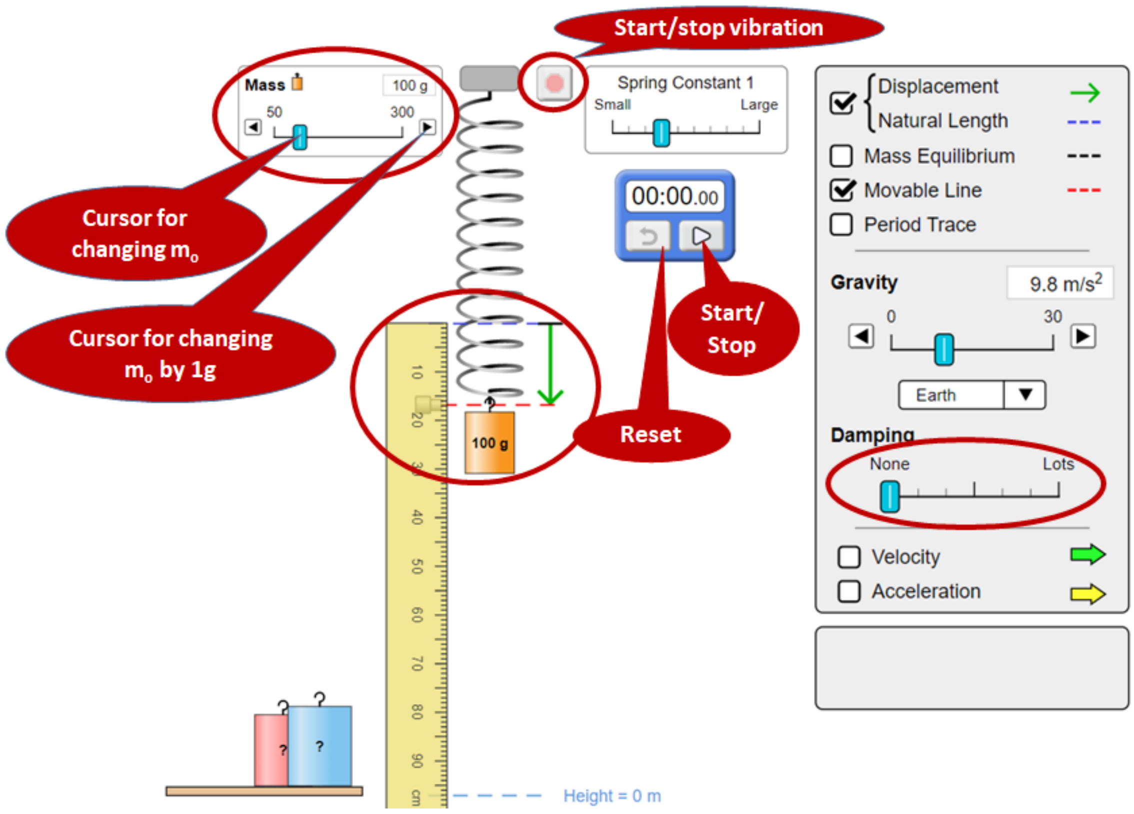

Fig. 3

5. Drag the mass m_o to the free end of the spring and click on  stop button to stop vibration of the spring (Fig. 3).
6. Measure extension (Δl) of the spring for the applied mass (Say, $m_o = 50$ g) using the scale and movable line.
7. Move the cursor at "**Damping**" to "**None**" (Fig. 3).
8. Stretch the mass downward to about 10 cm from the equilibrium position and allow the spring to vibrate. Measure time for 50 vibrations of the spring for the applied "m_o" using the stopwatch.
9. Change the "m_o" value with the cursor on the left (up) of the spring (Fig. 3).
10. Choose any 10 "m_o" values between 70 and 300 g (increase the mass by 25 g) and measure "Δl" of the spring. Record the data in Table.
11. Measure time for 50 vibrations for each "m_o" value. Record the data in Table.
12. Calculate time period for each "m_o" and record those in Table.
13. Draw a graph plotting "m_o" along X-axis and "Δl" along Y-axis. The plotted graph will be a straight line passing through the origin (Fig.1.4).
14. Calculate the slope of the m_o versus Δl graph. Use the slope in the equation (7) for determination of spring constant (k). Express "k" in C.G.S. unit.
15. Draw a graph plotting "m_o" along X-axis and square of the time period (T^2) along Y-axis. The graph will be a straight line intersecting the negative X-axis at a point (Fig. 5). Record the negative X intercept of the graph which will be the effective mass (m') of the spring.

Data collections:

Table: Data for measurements of extension and time period of a loaded spring.

Sl. No.	Mass on the spring, m_o (g)	Extension, Δl (cm)	Time for 50 vibrations, t (s)	Time period, T (s)	Square of time period, T^2 (s^2)
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

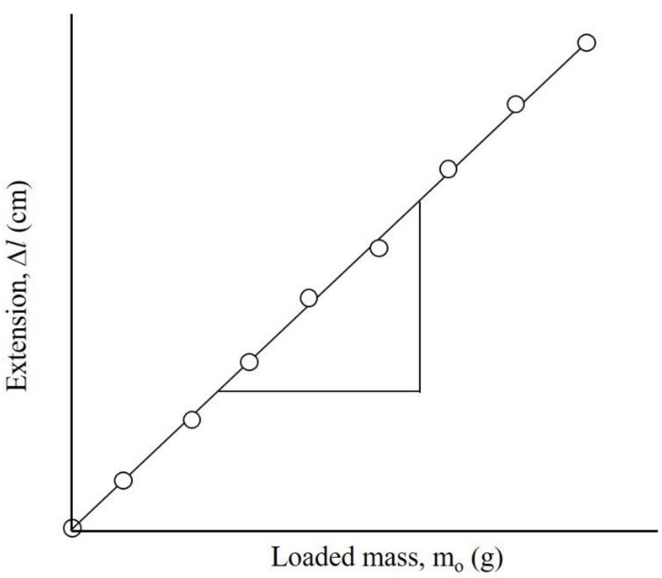

Fig. 4 A plot of mass versus extension of the loaded spring.

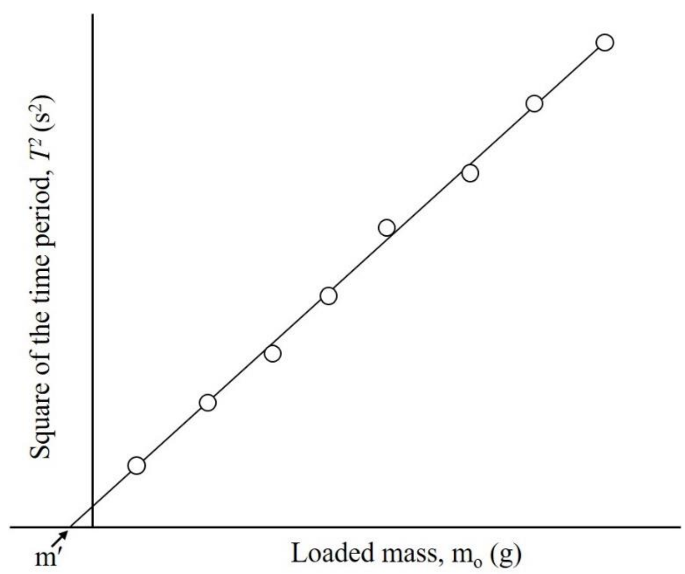

Fig. 5 A plot of mass versus T^2 of the loaded spring during vibration.

Calculations:

Slope of the m_o vs Δl graph =…………………………………………….

$k = \frac{g}{slope}$ =………………………………………………………………

X-intercept of the m_o vs T^2 graph =………………………………………

Results:

The spring constant of the chosen spring is k = …………………………

The effective mass of the vibrating spring is m' =………………………

Error analysis (if necessary):

Discussion:

Based on your understanding from this experiment, answer the following questions:

1) Which graph in this experiment represents Hooke's law? Mention how? Why the graph of T^2 as a function of m_o does not pass through the origin?
2) What will be the effect on the spring vibration if spring is changed, keeping applied load the same? How will be the effect on the experiment if it is done on the moon?
3) How does spring constant depend on the material and the length of the spring?
4) When does the mass-spring system attain (i) the maximum kinetic energy and (ii) the maximum potential energy during vibration? How does the total energy of the mass-spring system vary during free vibration?
5) What will be the nature of vibration if damping is included in the mass-spring experiment? If the mass-spring system is allowed to vibrate in a highly viscous fluid medium, what will be the nature of vibration of the system?
6) What will be the nature of vibration if the loaded spring is stretched not exactly along a vertical straight line path?