VL-W2: DETERMINATION OF ACCELERATION DUE TO GRAVITY '*g*' AND VERIFICATION OF CONSERVATION OF ENERGY OF A SIMPLE PENDULUM

Objectives:

- To study the periodic motion in a plane and investigating the relationship between the time period of a simple pendulum and its length.
- To determine the acceleration due to gravity learning about the conservation of energy.

System Requirement:

Computer (Desktop/Laptop), Operating system: Windows, pendulum-lab_en.html file, latest version of web browser (Example: Google Chrome).

Advise:

Students are advised to follow the **procedures written in this manual very strictly** while performing the experiment. D**o not try to explore anything else during the experiment.**

Theory:

An ideal pendulum is a point mass (m), called bob, suspended at one end of a massless string with the other end of the string fixed as shown in Fig. 1.

The motion of the system takes place in a vertical plane when the m is released from an initial angle θ. The angular amplitude θ is the angle that the string makes with the vertical direction.

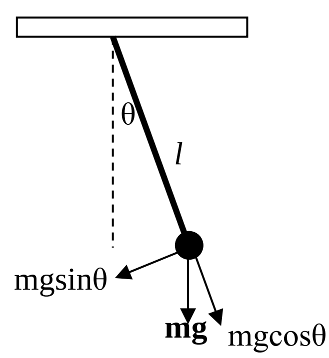


Fig. 1: A typical simple pendulum

The weight of the pendulum acts downward, and it can be resolved into two components. The component ($mgcos\theta$) is equal in magnitude to the tension in the string. The other component acts tangent to the arc along which the m moves. This component provides the force which drives the system. In equation form; the force (F) along the direction of motion is:

$$-F = mg\, sin\, \theta \qquad (1)$$

For small values of the initial angle θ, $sin\theta \approx tan\theta = x/l$, can be used in Eq. 1, and then becomes

$$F = -\frac{mgx}{l} \qquad (2)$$

where x is the displacement of m from the vertical position, l is length from the point of suspension to the center of the bob and g is the acceleration due to gravity.

Therefore, according to Newton's second law:

$$m\frac{d^2x}{dt^2} = -\frac{mgx}{l} \quad \Rightarrow \quad \frac{d^2x}{dt^2} + \frac{g}{l}x = 0 \qquad (3)$$

The vibration of a pendulum describes the simple harmonic motion, the period is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \Rightarrow \quad T^2 = \frac{4\pi^2}{g}l \qquad (4)$$

where T is the time period. There is a direct relationship between the angle θ and the velocity. Because of this, the mass does not affect the behavior of the pendulum and does not alter the period of the pendulum.

$$\text{Now,} \qquad g = \frac{4\pi^2}{\frac{T^2}{l}} \qquad (5)$$

The Law of Conservation of Energy: Energy cannot be created or destroyed, but is merely changed from one form into another. When no non-conservative forces (e.g., frictional forces) are present, the total mechanical energy of a pendulum is conserved, that is,

Total energy = KE + PE = constant

$$= \frac{1}{2}mv^2 + mgh = \text{constant.}$$

When the pendulum bob is suspended from a string, it will come to rest with the string at the vertical position (equilibrium). When displaced slightly and released, the pendulum will oscillate about the equilibrium position. During its swing, when the displacement is maximum, then the kinetic energy is zero (minimum) and potential energy is maximum. As the pendulum swings through the equilibrium position, the kinetic energy is a maximum and the potential energy is zero (minimum). At any point in its swing, the sum of the kinetic and potential energies remains constant.

Procedures:

(a) <u>The acceleration due to gravity *g* for earth:</u>

1. Unzip "**pendulum-lab_en**" file. Now open the "**pendulum-lab_en.html**" file with your web browser say using "google chrome".

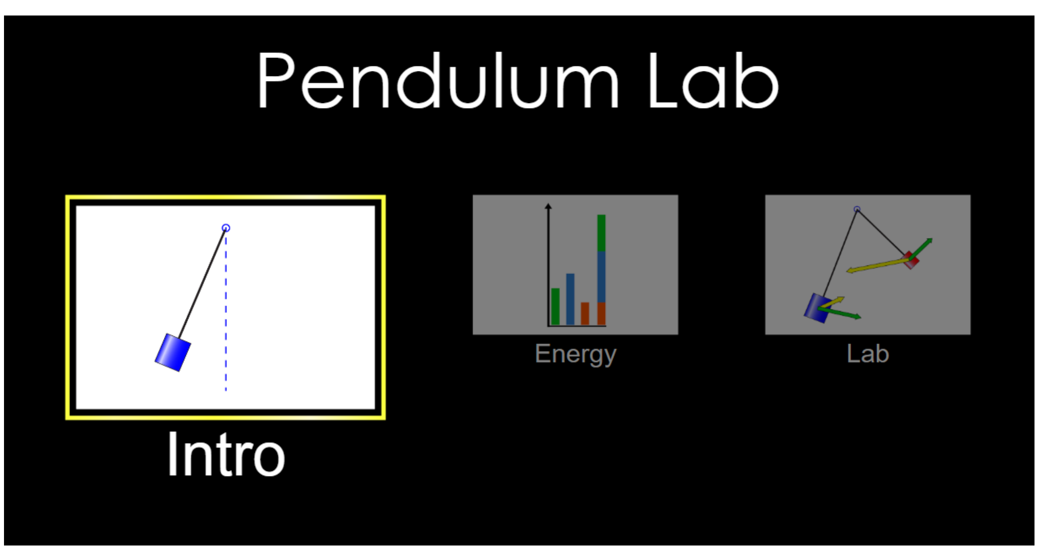


Fig. 2.

2. Click on '**Intro**' in Fig. 2 and find the lab screen as below (Fig. 3) and use the length controller to control the length of the pendulum (l), set l =1 m. Record the length in Table 1.

3. Control the angle (click on the mass for 1 kg and drag along the protractor to fix the angle), the angle must be very small (<10) and click on normal/slow mode of the simulation (Fig.3).

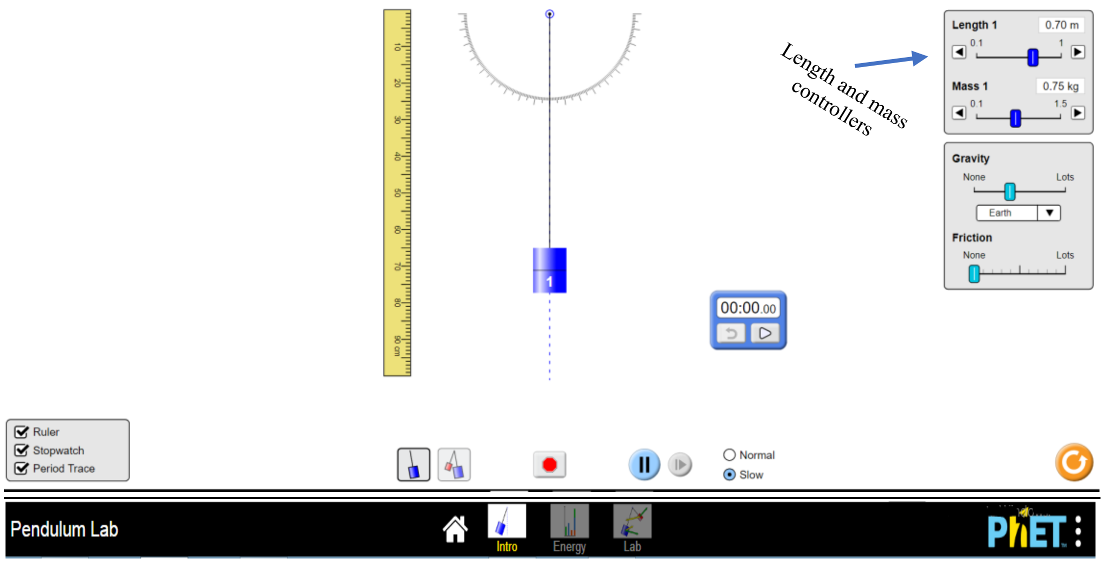
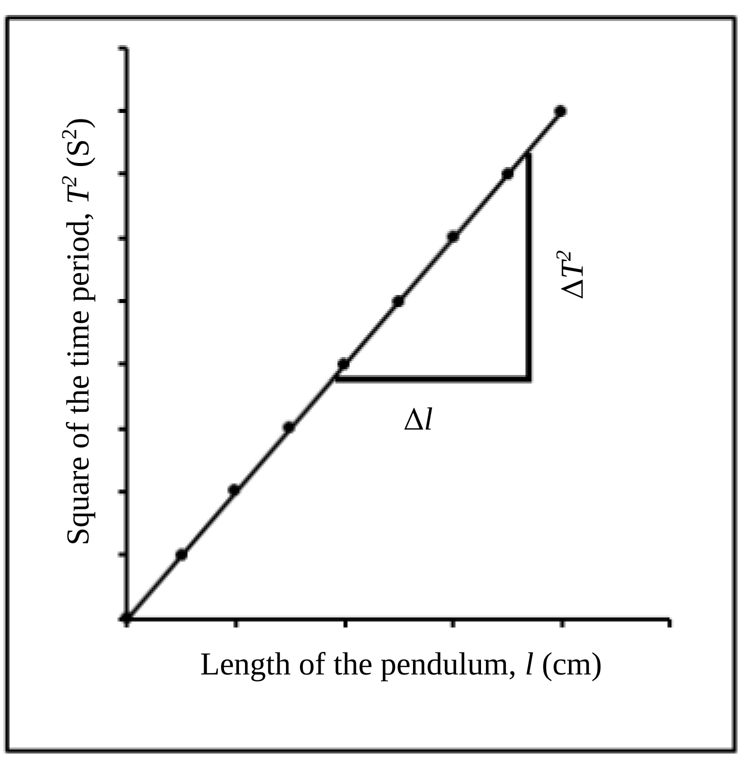


Fig. 3.

4. Click on **stop watch** (on the lower left corner of the lab screen) to measure the time required for 20 complete oscillations and determine the time period T.
5. Record the time period in Table 1.
6. Repeat the previous steps for different l as shown in the Table 1. Record your data.
7. Calculate the square of the time period (T^2).
8. Plot a graph of T^2 versus l (T^2 as the ordinate and l as the abscissa).
9. Use the slope of the line to calculate the acceleration due to gravity g using equation (5).



(b) <u>Conservation of energy</u>

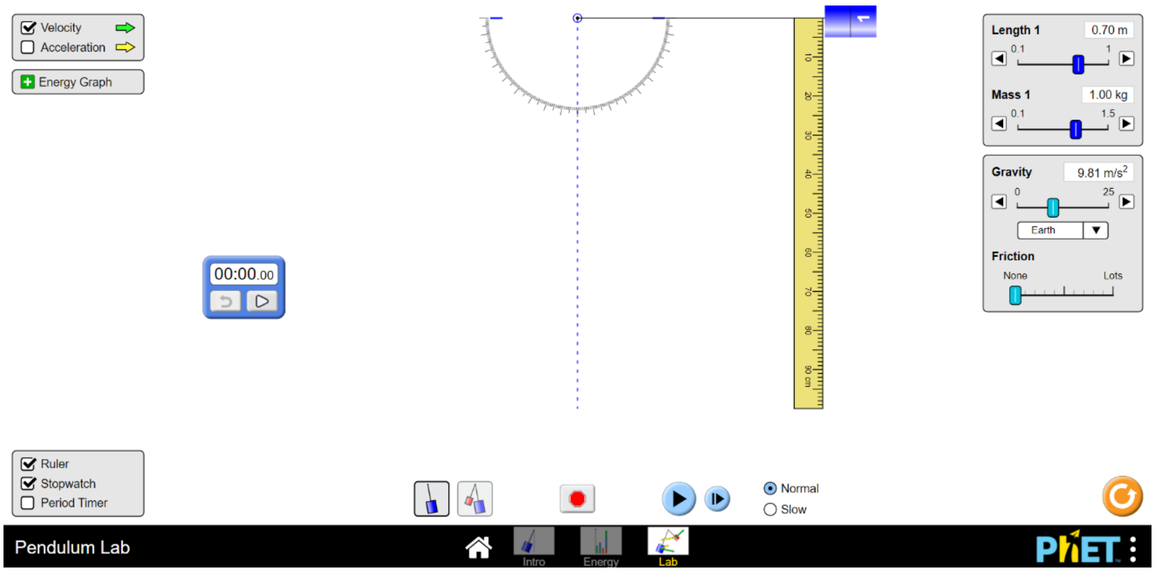


Fig. 4.

1. Select '**lab**' option in the bottom of Fig.3 and select one pendulum scenario as above (Fig.4).
2. Choose a fixed m (1 kg) and l (0.7 m) throughout the experiment.
3. Choose **no friction** initially.
4. Choose **Earth**.
5. Click **velocity, acceleration and energy graph** on the upper left corner of the lab screen and observe their variation.
6. Choose a **stopwatch** and a **ruler**.
7. Keep the bob (pendulum) at 90° position and measure the radius of the bob with the ruler (Fig. 4).
8. To determine the maximum speed, v_{max}, of the bob, choose "Normal" and displace the pendulum bob to the side equal to its radius as shown in Fig. 5 and measure the angle θ. Let the mass swing.
9. Measure the time required for 20 complete oscillations and determine the time period T.

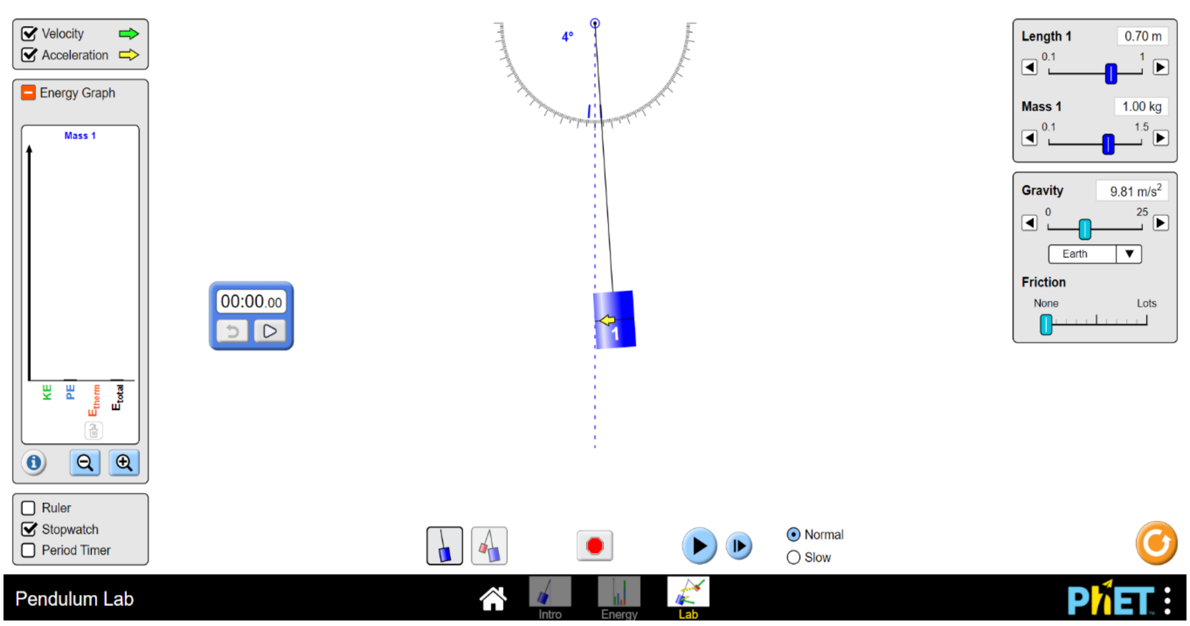


Fig. 5.

10. Calculate the height, h, of the pendulum bob shown in Fig. 5 by using the equation

$$h = l(1 - \cos\theta) \quad \ldots\ldots\ldots\ldots\ldots (6)$$

It is to be noted that when $\theta = 90°$ the pendulum is at its highest point.

i.e. $cos90° = 0$, and $h = l\,(1\text{-}0) = l$, and $PE = mgl(1 - cos\theta) = mgl$

When the pendulum is at its lowest point, $\theta = 0°$, $cos0° = 1$ and $h = l\,(1\text{-}1) = 0$, and

$$PE = mgl(1 - 1) = 0$$

At all points in-between the potential energy can be described using

$$PE = mgl(1 - cos\theta) \text{ .... } (7)$$

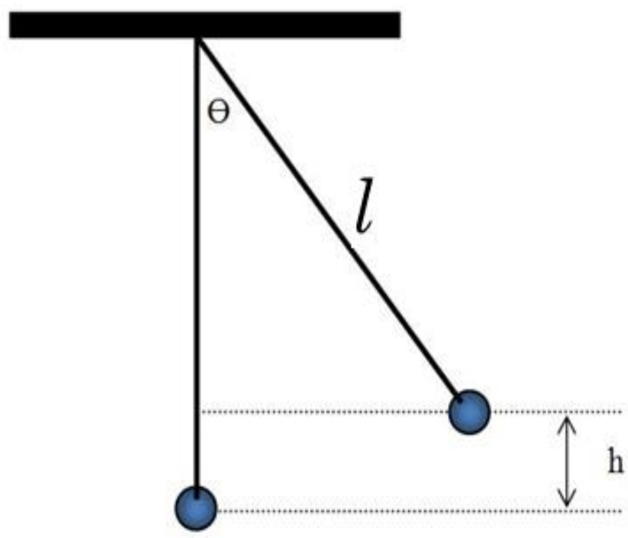


Fig. 6: Calculating height, *h*.

11. Calculate the maximum velocity V_{max}, of the pendulum bob shown in Fig. 5 by using the equation

$$y = A\, sin\, \omega t$$

$$\text{or, } v = \frac{dy}{dt} = A\omega\, cos\, \omega t$$

$$\text{or, } v_{max} = A\omega$$

$$\text{or, } v_{max} = A\frac{2\pi}{T}$$

Calculate the time period for $m = 1kg$, $l = 0.7m$ and $\theta = 4^0$

Using this height, the mass and maximum velocity of the pendulum bob, calculate the gravitational potential energy and kinetic energy for equilibrium and extreme positions and verify conservation of energy by equation (8).

$$KE_{extreme} + PE_{extreme} = KE_{equilibrium} + PE_{equilibrium}$$

$$\left[\frac{1}{2}mv^2 + mgl(1 - cos\theta)\right]_{extreme} = \left[\frac{1}{2}mv^2 + mgl(1 - cos\theta)\right]_{equilibrium} \quad (8)$$

Data Collections:

Table 1: Data for time period of simple pendulum.

Length of Simple Pendulum l (cm)	Time for 20 oscillations (s)	Time Period T(s)	T^2 (s^2)
1.0			
0.9			
0.8			
0.7			
0.6			
0.5			
0.4			
0.3			
0.2			
0.1			

Calculations:

(i) The acceleration due to gravity for earth

$$slope = \frac{\Delta T^2}{\Delta l}$$

$$g = \frac{4\pi^2}{slope}$$

(ii) Conservation of energy

$$\left[\frac{1}{2}mv^2 + mgl(1 - cos\theta)\right]_{\text{extreme}} = \left[\frac{1}{2}mv^2 + mgl(1 - cos\theta)\right]_{\text{equilibrium}}$$

Results;

The value of acceleration due to gravity, $g_{exp} =$

The energy of simple pendulum is conserved (or not conserved).

Error analysis:

$$\%\ of\ error = \frac{|Measured\ value - Standard\ value|}{Standard\ value} \times 100\% = \ldots.\%$$

Discussion:

Based on your understanding from this experiment, answer the following questions:

1) How would the period of a simple pendulum be affected if it were located on the moon instead of the earth?
2) Explain the effect of amplitude on the time period of oscillation.
3) Does changing the length of the pendulum affect the period of oscillation? Explain.
4) What happens to the period of a pendulum if the length is doubled?
5) Discuss the effect of the mass of the bob on the period of a simple pendulum. What would be the effect of replacing the steel ball with a wooden ball, a lead ball, and a ping pong ball of the same mass?
6) Discuss how the energy of the pendulum varies at different position during oscillation.
7) Compare the mechanical energy of the pendulum at large and small amplitudes.